Study of CP Asymmetry in the Neutral B Meson Decays to Two Charged Pions

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Abstract

We present the measurement of CP-violating asymmetries in $B^0 \to \pi^+\pi^-$ decays based on a 78 fb⁻¹ data sample collected at the $\Upsilon(4S)$ resonance with the Belle detector at the KEKB asymmetric-energy e^+e^- collider. We fully reconstruct one neutral B meson as a $B^0 \to \pi^+\pi^-$ CP eigenstate and identify the flavor of the accompanying Bmeson from its decay products. From the asymmetry in the distribution of the time intervals between the two B meson decay points, we obtain the CP-violating asymmetry parameters $\mathcal{A}_{\pi\pi} = +0.77 \pm 0.27(\text{stat}) \pm 0.08(\text{syst})$ and $\mathcal{S}_{\pi\pi} = -1.23 \pm 0.41(\text{stat}) \stackrel{+0.08}{_{-0.07}}(\text{syst})$, where the statistical uncertainties are determined from Monte Carlo pseudo-experiments. We rule out the CP-conserving hypothesis, $\mathcal{A}_{\pi\pi} = \mathcal{S}_{\pi\pi} = 0$, at a 99.93% confidence level. We discuss how these results constrain the value of CKM angle ϕ_2 .

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Chapter 1

Introduction

One of the most important concepts in physics is the symmetry or invariance under the operation in space and time, here the symmetry strongly relates to the conservation laws. The symmetries under the basic transformations, such as parity (P), charge-conjugation (C), and time-reversal (T) transformations, were believed until the middle of the 20th century. In 1950s, the parity violation was discovered in the β -decays, and the charge conjugation was also considered to be violated in the weak interaction because of the absence of the right-handed neutrino. However the product of CP had been thought to be conserved. Then the subsequent discovery of CP violation gave a deep impact to the modern particle physics.

The CP violation is expected to relate on the basic principle of the nature. For example, if we believe that there were the same amount of *matter* and *anti-matter* at the beginning of the universe as explained in the *Big-Bang* theory, the existence of CP violation is one of the key issues to explain why the universe we live in consists predominantly of the matter, as A. D. Sakharov pointed out [1].

The CP violation was first observed in the neutral K-meson system in 1964 by J.H. Christenson et al. [2]. This discovery initiated many theoretical efforts to understand the CP violation phenomenon. In 1973, M. Kobayashi and T. Maskawa proposed a theory of quark mixing which can introduce the CP violation within the framework of the *Standard Model* of elementary particle physics [3]. They noted that one or more irreducible complex phases in the quark-flavor mixing matrix can introduce the CP violation, and pointed out that this requirement was satisfied if there exist at least three generations of quarks, i.e. six quarks, even only three quarks, up (u), down (d), and strange (s) were known at that time. In 1970s, the fourth and the fifth quarks, charm (c) and bottom (b), were discovered. The observation of the sixth quark, top (t) quark, in 1995 established the *Kobayashi-Maskawa model* (KM model) as an essential part of the Standard Model.

It is not known, however, whether the KM model explanation is quantitatively correct, though the model is elegant and economical to explain the CP violation. Therefore precise measurements of the magnitude and the phase of the quark-mixing matrix, which is called Cabibbo-Kobayashi-Maskawa (CKM) matrix, and the consistency tests are important. The relation between the CKM matrix elements is presented by the *unitarity triangle*. The measurements of sides and angles of the unitarity triangle give the tests of the KM model, since the KM model provides the definitive predictions for three angles of the unitarity triangle, ϕ_1 , ϕ_2 and ϕ_3 .

In 1980, A. B. Carter, A. I. Sanda and I. I. Bigi pointed out that the sizable CP violation can be observed in the *B*-meson system in the framework of the Standard Model. They also predicted it is possible to measure the angles of the unitarity triangle directly from the difference of the partial time-dependent decay rates between B^0 and \overline{B}^0 to the same CP eigenstate. Subsequent observations of the long *B*-meson lifetime [4,5] and the large flavor-mixing in the neutral *B* meson system [6] indicated that it would be feasible to carry out the several measurements of CP violation in the *B*-meson system. The experimental method using an asymmetric e^+e^- collider at $\Upsilon(4S)$ was proposed [7], and two *B*-factory accelerators were constructed in Japan and USA.

In 2001, the CP violation in the *B*-meson system was observed in *B*-factory experiments [8,9] as predicted by the KM model. Here ϕ_1 was measured to be consistent with the KM model prediction using the decay modes $B^0 \rightarrow J/\psi K^0$ and so on. Currently the measurements draw a consistent picture that the CP violation is induced by the quark-flavor mixing as predicted by the KM model. Now *B* physics is in the next stage to deal with the precise determination and test of the KM model, relating the CP violation, in both experimentally and theoretically.

In this thesis, we present the study of CP violation in $B^0 \to \pi^+\pi^-$ decays. This decay mode is sensitive to the angle ϕ_2 of the unitarity triangle through a measurement of the CP violating asymmetry, then gives the other test of the KM model than the ϕ_1 measurement with $B^0 \to J/\psi K^0$ decays. Although the ϕ_2 extraction from the $B^0 \to \pi^+\pi^-$ decays alone has the large theoretical uncertainty because the decay receives contributions from both the $b \to u$ tree diagram and the $b \to dg$ penguin diagram, the measurement is expected to play an important role to understand the *CP*-violation mechanism, by observing direct *CP* violation. The direct *CP* violation is the other type *CP* violation than the mixing-induced *CP* violation which is observed in $B^0 \to J/\psi K^0$ decays.

In the $B^0 \to \pi^+\pi^-$ decays, the *CP* violation is observed as the time-dependent decay-rate asymmetry in B^0 and \overline{B}^0 mesons. Experimentally, the proper-time difference Δt between the two neutral *B* mesons, where one decays to $\pi^+\pi^-$ (B_{CP}) and the other decays to anything (B_{tag}), is measured. The *CP* violation is extracted from the difference between the Δt distributions for $B_{tag} = B^0$ and $B_{tag} = \overline{B}^0$.

The study is performed at the KEK *B*-factory experiment, which consists of the accelerator called KEKB and the multi-purpose detector called Belle. KEKB is an asymmetric energy e^+e^- collider operated on the $\Upsilon(4S)$ resonance and it produces a huge number of *B* mesons with clean final states. Because produced *B* mesons are boosted, a time-dependent decay rate can be measured using the information of the flight lengths of *B* mesons. The Belle detector is designed to be suitable for the *CP* violation measurements: it provides the precise determination of the decay points of *B* mesons, the good particle-identification capability to identify the flavor of *B* mesons, i.e. B^0 or \overline{B}^0 , the data acquisition system withstand the high event rate to achieve the high statistics, and so on.

Currently the *B*-factory experiment is unique to carry out the study of *CP* violation using $B^0 \to \pi^+\pi^-$ decays. Because the final state has the low particle multiplicity, the signals are hidden by huge backgrounds in hadron colliders. Moreover, the branching fraction of $B^0 \to \pi^+\pi^-$ is quite small of $\mathcal{O}(10^{-6})$. Thus, the high luminosity e^+e^- collider is required.

In this thesis, we present the first evidence of CP violation in $B^0 \to \pi^+\pi^-$ decays. The outline of this thesis is as follows: Theoretical description of CP violation and the experimental consideration to measure the CP violation in the neutral *B*-meson system are provided in Chapter 2. Experimental apparatus used in this study is described in Chapter 3. The reconstruction procedure of the candidates for $B^0 \to \pi^+\pi^-$ decays is explained in Chapter 4. The measurement of CP asymmetry in $B^0 \to \pi^+\pi^-$ decays is described in Chapter 5. Finally the estimation of the statistical significance, the extraction of the ϕ_2 information and the conclusion of this thesis are written in Chapter 6.

Chapter 2

CP violation in the B meson system

2.1 Introduction of the *CP* violation

In this chapter, the basic theory of CP violation and the experimental consideration to measure the CP violation in the neutral B meson system are described. First, we show the definition of the CP transformation. Then we review the CP violation in the K-meson system and the quark mixing in the weak interaction. We explain the Kobayashi-Maskawa model based on the quark mixing. We discuss the $B^0-\overline{B}^0$ mixing and the CP violation in Bmeson system based on the KM model, and show how to measure the CP violation in the B factory experiment. Finally, we review the present experimental status of the test of the KM mechanism.

2.1.1 Parity, Time reversal and Charge Conjugation Operations

A parity transformation, P, reverses three-dimensional coordinates:

$$P|f(\boldsymbol{p},\boldsymbol{s})\rangle = \eta_P|f(-\boldsymbol{p},\boldsymbol{s})\rangle, \qquad (2.1)$$

where $|f(\mathbf{p}, \mathbf{s})\rangle$ represents the particle f with a momentum of \mathbf{p} and a spin of \mathbf{s} , and η_P is a phase of parity transformation. Since P^2 is an identical operator, η_P^2 is 1, and thus $\eta_P = \pm 1$. The sign of the η_P is chosen arbitrary as far as the definition is consistent through the discussion.

A time reversal transformation, T, inverses the time coordinates:

$$T|f(\boldsymbol{p},\boldsymbol{s})\rangle = \eta_T|f(-\boldsymbol{p},-\boldsymbol{s})\rangle,$$
(2.2)

where η_T is a phase and $\eta_T^2 = 1$.

A charge conjugation, C, changes the signs of internal charges such as electric charge, baryon number and so on. This operation replaces particles with their anti-particles (and vice versa):

$$C|f(\boldsymbol{p},\boldsymbol{s})\rangle = \eta_C |\bar{f}(\boldsymbol{p},\boldsymbol{s})\rangle, \qquad (2.3)$$

where η_C is a phase and $\eta_C^2 = 1$.

If a state is an eigenstate of these operators, it has an eigenvalue of ± 1 . We call the eigenstate with an eigenvalue of +1 (-1) even (odd) state. Particles like mesons, baryons and gauge bosons may have *intrinsic* eigenvalues of above symmetries.

It used to be naively believed that all are conserved under these transformations, C, T and P. However, in 1956, T. Lee and C. Yang questioned the assumption of the parity conservation [10], and the subsequent experiments by C. Wu et al. and by R. Garwin et al. in 1957 independently demonstrated the violation of the P-invariance and the C-invariance in weak decays of nuclei and of muons [11, 12]. It was found in successive experiments that the parity is maximally violated in the weak interaction. The neutrinos are completely polarized, i.e. a neutrino has a helicity¹ of -1 (left-handed), and an anti-neutrino has a helicity of +1 (right-handed). A left-handed neutrino applied P or C operation becomes the unphysical state, as illustrated in Figure 2.1. In this example, however, the combined operation CP to a left-handed neutrino is not forbidden.

The combined operation CPT has special importance in the quantum field theory. General principles of relativistic field theory require the symmetry under the CPT transformation (CPT theorem). The CPT theorem assures the equality of the masses, lifetimes, and magnitudes of charge between particle and anti-particles. Up to now, no CPT violation has been observed experimentally.

 $^{^{1}}$ the sign of spin along the direction of motion



Figure 2.1: Mirror image of a left-handed neutrino under C, P and CP transformation, where arrows indicate the direction of motion.

2.1.2 The Discovery of CP violation in the K Meson System

Neutral K mesons, which has a non-zero quantum number, *strangeness*, are produced as the eigenstates of the interaction, $|K^0\rangle$ and $|\overline{K}^0\rangle$. M. Gell-Mann and A. Paris predicted the K^0 - \overline{K}^0 mixing. They pointed out that K^0 (\overline{K}^0) changes to the mixture of CP eigenstates,

$$\left|K_{1}^{0}\right\rangle = \left(\left|K^{0}\right\rangle - \left|\overline{K}^{0}\right\rangle\right)/\sqrt{2} \tag{2.4}$$

$$|K_2^0\rangle = (|K^0\rangle + |\overline{K}^0\rangle)/\sqrt{2}, \tag{2.5}$$

where CP eigenvalues of $|K_1^0\rangle$ and $|K_2^0\rangle$ are +1 and -1, respectively, with the notation of $CP|K^0\rangle = -|\overline{K}^0\rangle$. They also argued on the lifetimes of K_1^0 and K_2^0 . Assuming that the CP symmetry is conserved, only K_1^0 can decay into the 2π final state, and K_2^0 decays into 2π is forbidden because CP eigenvalue of the 2π state is +1. Then the leading non-leptonic decay channel for K_2^0 is $K_2^0 \to 3\pi$, which is suppressed due to the very restricted phase space. Thus, the lifetime for K_2^0 is expected to be long respect to K_1^0 . According to their prediction, in 1965, K. Lande et al. [13] discover the long-lived particle. Since the lifetimes were quite different, ~90ps and ~52 ns, K_1^0 and K_2^0 were referred to as K_S and K_L , respectively.

Whereas CP conservation was believed at that time, J. Christenson et al. discovered CP-violating $K_L \to \pi^+\pi^-$ decays in 1964 [2]. Even the branching fraction of $K_L \to \pi^+\pi^-$ is small ($\mathcal{O}(10^{-3})$), this indicates mass eigenstates, the K_S and K_L , are not CP eigenstates and should be rewritten as

$$|K_S\rangle = (|K_1^0\rangle + \epsilon_m |K_2^0\rangle)/\sqrt{1 + |\epsilon_m|^2}$$
(2.6)

and

$$|K_L\rangle = (\epsilon_m |K_1^0\rangle + |K_2^0\rangle)/\sqrt{1 + |\epsilon_m|^2}.$$
(2.7)

Subsequent observation of $K_L \to \pi^0 \pi^0$ decays [14, 15], and charge asymmetries in $K_L \to \pi^{\pm} e^{\pm} \nu$ [16] and $K_L \to \pi^{\pm} \mu^{\pm} \nu$ [17] confirmed the *CP* violation in the neutral *K*-meson system.

2.2 The standard model and the quark mixing.

Since the semi-leptonic decay, which changes the strangeness, such as $\Sigma^- \to ne^-\bar{\nu}_e$ is suppressed by a factor ~20 compared with $n \to pe\bar{\nu}_e$ and the β -decay of neutron is also suppressed by a few% respect to $\mu \to \bar{\nu}_\mu e\bar{\nu}_e$, the universality of the coupling constant of weak interactions looks violated outwardly. In 1963, N. Cabibbo proposed the model that explained these differences of the decay rate with a single coupling constant [18]. In 1964, M. Gell-Mann and G. Zweig proposed the quark model [19, 20] and introduced three flavors of quarks, u (up), d (down) and s (strange). The Cabibbo model was interpreted in the context of the quark model. He proposed that the d and s quark states participating a weak interaction are not pure flavor eigenstates, but the mixture of them; i.e. the state of

$$d'\rangle \equiv \cos\theta_c |d\rangle + \sin\theta_c |s\rangle \tag{2.8}$$

is coupled with u quark via a unique coupling constant of weak interactions, where θ_c is called *Cabibbo Angle*. This model represents the difference of decay rates of semi-leptonic decays and the Cabibbo angle is obtained as $\sin \theta_c \simeq 0.22$ ($\cos \theta_c \simeq 0.98$). The Cabibbo scheme based on the three-quark model does not forbid the neutral

2.3. KOBAYASHI-MASKAWA MODEL

current to change the strangeness, such as $K^0 \to \mu^+ \mu^-$, but is stringently suppressed in reality. In 1970, S. L. Glashow, J. Iliopoulos, and L. Maiani proposed to introduce the fourth quark, c(charm) in order to explain this suppression (GIM mechanism) [21]. In this model, the quark-doublet state of

$$\begin{pmatrix} d'\\s' \end{pmatrix} \equiv \begin{pmatrix} \cos\theta_c & \sin\theta_c\\ -\sin\theta_c & \cos\theta_c \end{pmatrix} \begin{pmatrix} d\\s \end{pmatrix}$$
(2.9)

couples with the quark doublet (u, c) via a unique coupling constant. For example, the two diagrams, which are shown in Figure 2.2 contribute to $K^0 \to \mu^+ \mu^-$, and each diagram cancels the other out.

In 1974, the existence of the charm quark was implied by the J/ψ meson discovery [22, 23], and the GIM mechanism was confirmed².



Figure 2.2: Two diagrams contributing the decay $K^0 \rightarrow \mu^+ \mu^-$.

2.3 Kobayashi-Maskawa model

In 1973, M. Kobayashi and T. Maskawa proposed that the existence of at least three quark generations can explain the CP violation within the framework of the Standard Model [3].

2.3.1 Cabibbo-Kobayashi-Maskawa Matrix

They extend the framework of the quark mixing proposed by Cabibbo-GIM from 2-generations, (u, d) and (c, s), to general N generations. Then the quark-mixing matrix in Equation 2.9 is extended to $N \times N$ complex matrix, V_{ij} , which has $2N^2$ real parameters. Since the matrix is a unitarity matrix, the conditions of $\sum_j V_{ij}V_{jk}^* = \delta_{ik}$ are required, where δ_{ik} is Kronecker's δ . Thus, the number of the free parameter of the matrix is reduced to N^2 . Since the phase of the quark fields can be rotated arbitrarily except an overall phase, (2N - 1) relative phases can be removed from the matrix, and then the number of free parameter becomes $(N - 1)^2$. On the other hand, a general $N \times N$ orthogonal matrix has N(N - 1)/2 rotational Euler angle. Thus, (N - 2)(N - 1)/2 parameters corresponding to irreducible *phase* remain in the matrix V.

To summarize, the generalized quark-mixing matrix for N generations consists of N(N-1)/2 rotational Euler angles and (N-1)(N-2)/2 irreducible phases independently. In the two quark-generations model, the quarkmixing matrix has one angle and no complex phase as in the Cabibbo-GIM theory described in Section 2.2. If there exists three quark-generations, the quark-mixing matrix has three angles and a complex phase. Kobayashi and Maskawa pointed out that the complex phase, which causes the CP violation, can be introduced to the Lagrangian of the weak interaction via the quark-mixing matrix if there are at least three quark generations. Subsequent discoveries of c, b (bottom/beauty) and t (top) quarks made the Kobayashi-Maskawa mechanism to be incorporated into the standard model [24–26].

The 3×3 quark-mixing matrix in the standard model is now called the Cabibbo-Kobayashi-Maskawa (CKM) matrix, V_{CKM} :

$$V_{\rm CKM} \equiv \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix}, \qquad (2.10)$$
$$\begin{pmatrix} d' \\ s' \\ b' \end{pmatrix} = V_{\rm CKM} \begin{pmatrix} d \\ s \\ b \end{pmatrix}. \qquad (2.11)$$

²Note that c quarks was discovered after the Kobayashi-Maskawa model, which is described in Section 2.3, was proposed.

L. Wolfenstein proposed to parameterize the CKM matrix in the form of an expansion in $\lambda \equiv \sin \theta_c \simeq 0.22$ as:

$$V_{\rm CKM} = \begin{pmatrix} 1 - \lambda^2/2 & \lambda & A\lambda^3(\rho - i\eta) \\ -\lambda & 1 - \lambda^2/2 & A\lambda^2 \\ A\lambda^3(1 - \rho - i\eta) & -A\lambda^2 & 1 \end{pmatrix} + \mathcal{O}(\lambda^4) , \qquad (2.12)$$

where A, ρ and η are real parameters of order of unity [27].

The Unitarity Triangle 2.3.2

 $\mathcal{O}(1)$

The unitarity of the CKM matrix leads to the relations:

$$\sum_{j} V_{ij} V_{jk}^* = 0 \qquad (i \neq k).$$
(2.13)

Since the CKM matrix elements are complex, these relations imply that they form triangles in a complex plane, such as shown in Figure 2.3(a). These triangles are often referred to as the unitarity triangles. There are six unitarity triangles. The six unitarity conditions are explicitly written as

$$\underbrace{V_{ud} V_{us}^*}_{\mathcal{O}(\lambda^1)} + \underbrace{V_{cd} V_{cs}^*}_{\mathcal{O}(\lambda^1)} + \underbrace{V_{td} V_{ts}^*}_{\mathcal{O}(\lambda^5)} = 0, \qquad (2.14)$$

$$\underbrace{\underbrace{V_{us}}_{\mathcal{O}(\lambda^4)}}_{\mathcal{O}(\lambda^4)} + \underbrace{\underbrace{V_{cs}}_{\mathcal{O}(\lambda^2)}}_{\mathcal{O}(\lambda^2)} + \underbrace{\underbrace{V_{ts}}_{ts}}_{\mathcal{O}(\lambda^2)} = 0, \qquad (2.16)$$

$$\underbrace{V_{cd}}_{\mathcal{O}(\lambda^4)} \underbrace{V_{td}}_{\mathcal{O}(\lambda^4)}^* + \underbrace{V_{cs}}_{\mathcal{O}(\lambda^2)} \underbrace{V_{tb}}_{\mathcal{O}(\lambda^2)}^* + \underbrace{V_{cb}}_{\mathcal{O}(\lambda^2)} \underbrace{V_{tb}}_{\mathcal{O}(\lambda^2)}^* = 0, \qquad (2.17)$$

$$\underbrace{V_{ud}}_{\mathcal{O}(\lambda^3)} \underbrace{V_{td}}_{\mathcal{O}(\lambda^3)} + \underbrace{V_{us}}_{\mathcal{O}(\lambda^3)} \underbrace{V_{tb}}_{\mathcal{O}(\lambda^3)} + \underbrace{V_{ub}}_{\mathcal{O}(\lambda^3)} \underbrace{V_{tb}}_{\mathcal{O}(\lambda^3)} = 0$$
(2.18)

and

$$\underbrace{V_{ud} \quad V_{ub}}_{\mathcal{O}(\lambda^3)} + \underbrace{V_{cd} \quad V_{cb}}_{\mathcal{O}(\lambda^3)} + \underbrace{V_{td} \quad V_{tb}}_{\mathcal{O}(\lambda^3)} = 0, \tag{2.19}$$

where the magnitude of each term is evaluated using the Wolfenstein parameterization. The 4 unitarity triangles have extremely squashed shapes because the magnitudes of the terms in the unitarity conditions are not balanced. On the other hand, since all the terms in Equation 2.18 and 2.19 are the same order of $\mathcal{O}(\lambda^3)$, the angles in two unitarity triangles that correspond to these equations expected to be equally large. Equation 2.18 and 2.19 are related to b and t quark decay, respectively. Thus, it is expected to observe relatively larger CP violations in B meson system than the K meson system. A large CP violation is expected in t quark system also, but it is difficult to investigate the top quark system experimentally due to its large mass. B meson system is therefore the most suitable to examine the Kobayashi-Maskawa mechanism.

Figure 2.3(a) shows the unitarity triangle, which corresponds to Equation 2.19. It is a test of the Kobayashi-Maskawa mechanism whether the sides and angles of the unitarity triangle, which are determined experimentally, form a closed triangle, or not. Especially, the CP-conservation is violated if the area of the triangle does not vanish. This requires all of the three angles in the unitarity triangle:

$$\phi_1 \equiv \pi - \arg\left(\frac{-V_{td}V_{tb}^*}{-V_{cd}V_{cb}^*}\right),\tag{2.20}$$

$$\phi_2 \equiv \arg\left(\frac{V_{td}V_{tb}^*}{-V_{ud}V_{ub}^*}\right),\tag{2.21}$$

and

$$\phi_3 \equiv \arg\left(\frac{V_{ud}V_{ub}^*}{-V_{cd}V_{cb}^*}\right) \tag{2.22}$$

should be non-zero values.

It is convenient to rescale the unitarity triangle by dividing all the sides by $V_{cd}V_{cb}^*$, as shown in Figure 2.3(b). The rescaled triangle has the vertices at (0,0), (1,0) and $(\bar{\rho},\bar{\eta})$, where $\bar{\rho}$ and $\bar{\eta}$ are related to ρ , η and λ in the Wolfenstein parameterization as

$$\bar{\rho} = (1 - \lambda^2/2)\rho \tag{2.23}$$
 and

$$\bar{\eta} = (1 - \lambda^2/2)\eta, \qquad (2.24)$$

respectively.



Figure 2.3: The unitarity triangle of the CKM matrix (left) and its rescaled form in ρ - η plane (right).

2.4 CP violation in the B meson system

In 1980, A. B. Carter, A. I. Sanda and I. I. Bigi pointed out that the Kobayashi-Maskawa mechanism indicates the possibility of the sizable CP violation in the B meson system [28–30]. In the K meson system, the CP-violating phase appears through the off-shell transitions to heavy flavors, such as $s \to W^-c, W^-t$, which occur during $K^0-\overline{K}^0$ mixing, and the order of CP asymmetry is about 10^{-3} . On the other hand, in the B meson system, the CP-violating phase also enters in the on-shell transitions, such as $b \to W^-c$, which make up the decay cascades of the b quark. The on-shell transitions can produce CP asymmetry of order $10^{-1} - 10^{-2}$.

Subsequent observations of long B mesons lifetimes [4,5] and large mixing in the neutral B meson system [6] indicated that it would be feasible to carry out the measurement of CP violation in asymmetric e^+e^- collider [7].

In this section, the phenomenology of the time evolution and the CP violation in the B meson system is described.

2.4.1 $B^0 - \overline{B}^0$ mixing

 B^0 and $\overline{B}{}^0$ can mix through the second order weak interactions described with the diagrams shown in Figure 2.4. Thus, the time-dependent wave function of a neutral B meson is written as a linear combination of the B^0 and $\overline{B}{}^0$ states:

$$|B(t)\rangle = \alpha(t)|B^{0}\rangle + \beta(t)|\overline{B}^{0}\rangle, \qquad (2.25)$$

where we use the notation of $CP|B^0\rangle = -|\overline{B}^0\rangle$. The phenomenological time-dependent Schrödinger equation:

$$i\frac{\partial}{\partial t}|B(t)\rangle = \mathcal{H}|B(t)\rangle$$
(2.26)

is expanded as

$$i\frac{\partial}{\partial t} \begin{pmatrix} \alpha(t) \\ \beta(t) \end{pmatrix} = \begin{pmatrix} M_0 - i\Gamma_0/2 & M_{12} - i\Gamma_{12}/2 \\ M_{12}^* - i\Gamma_{12}^*/2 & M_0 - i\Gamma_0/2 \end{pmatrix} \begin{pmatrix} \alpha(t) \\ \beta(t) \end{pmatrix},$$
(2.27)

where the CPT theorem is assumed. M in the diagonal term is the mass of the flavor eigenstate of B^0 and \overline{B}^0 , and Γ is their width. The off-diagonal elements are responsible for the $B^0\overline{B}^0$ transition. The eigenvalues are given as

$$\mu_{\pm} = M_0 - i\Gamma_0/2 \pm \sqrt{(M_{12} - i\Gamma_{12}/2)(M_{12}^* - i\Gamma_{12}^*/2)}, \qquad (2.28)$$

(2.39)

which correspond to the eigenvectors of

$$|B_{\pm}\rangle = p|B^{0}\rangle \pm q|\overline{B}^{0}\rangle, \qquad (2.29)$$

$$\frac{q}{p} = \sqrt{\frac{M_{12}^2 - i\Gamma_{12}^2/2}{M_{12} - i\Gamma_{12}/2}},$$
(2.30)

where p and q are normalized as $|p|^2 + |q|^2 = 1$. In the B_d^0 meson system, the standard model predicts [31–34]

$$\left|\frac{\Gamma_{12}}{M_{12}}\right| \simeq \frac{3\pi m_b^2}{2m_t^2} \sim \mathcal{O}(10^{-2}) \ll 1, \tag{2.31}$$

where m_b and m_t are the mass of b and t quarks, respectively. Thus, p/q is approximated as

$$\frac{p}{q} \simeq \sqrt{\frac{M_{12}^*}{M_{12}}} = \frac{V_{tb}^* V_{td}}{V_{tb} V_{td}^*},\tag{2.32}$$

and, |p/q| = 1. The eigenvector $|B_{-}\rangle$ ($|B_{+}\rangle$) corresponds to the mass eigenstate $|B_{\rm H}\rangle$ ($|B_{\rm L}\rangle$) with a eigenvalue of $\mu_{\rm H} = \mu_{-}$ ($\mu_{\rm L} = \mu_{+}$). The time evolutions of mass eigenstates are expressed as:

$$|B_{\rm H}(t)\rangle = \exp\left(-i\mu_{\rm H}t\right)|B_{\rm H}\rangle \equiv \exp\left[-i(m_{\rm H} - i\Gamma_{\rm H}/2)t\right]|B_{\rm H}\rangle$$
(2.33)

and

$$|B_{\rm L}(t)\rangle = \exp\left(-i\mu_{\rm L}t\right)|B_{\rm L}\rangle \equiv \exp\left[-i(m_{\rm L} - i\Gamma_{\rm L}/2)t\right]|B_{\rm L}\rangle,\tag{2.34}$$

where the difference of mass and width of the two mass eigenstates are given as:

$$\Delta m \equiv m_{\rm H} - m_{\rm L} = -2 \text{Re} \sqrt{(M_{12} - i\Gamma_{12}/2)(M_{12}^* - i\Gamma_{12}^*/2)}$$
(2.35)

and

$$\Delta \Gamma \equiv \Gamma_{\rm L} - \Gamma_{\rm H} = -4 \operatorname{Re}(M_{12} \Gamma_{12}) / \Delta m_d, \qquad (2.36)$$

respectively. The time evolution of the state $|B^0(t)\rangle$ ($|\overline{B}^0(t)\rangle$) which is initially the eigenstate of the weak interaction, $|B^0\rangle$ ($|\overline{B}^0\rangle$), is obtained from Equations 2.33 and 2.34, and the relations of

$$|B^{0}\rangle = (|B_{\rm H}\rangle + |B_{\rm L}\rangle)/2p \tag{2.37}$$

and

$$\overline{B}^{0}\rangle = -(|B_{\rm H}\rangle - |B_{\rm L}\rangle)/2q.$$
(2.38)

Thus,

$$B^{0}(t)\rangle = g_{+}(t)|B^{0}\rangle + \frac{q}{p}g_{-}(t)|\overline{B}^{0}\rangle$$
(2.40)

and

$$\left|\overline{B}^{0}(t)\right\rangle = g_{+}(t)\left|\overline{B}^{0}\right\rangle + \frac{p}{q}g_{-}(t)\left|B^{0}\right\rangle, \qquad (2.41)$$

$$g_{\pm}(t) \equiv \frac{1}{2} \exp\left[-i(m_{\rm L} - i\Gamma_{\rm L}/2)t\right] \cdot \left\{1 \pm \exp\left[-i(\Delta m_d + i\Delta\Gamma/2)t\right]\right\}$$
(2.42)

$$= \frac{1}{2} \exp\left(-imt\right) \exp\left(-\Gamma t/2\right) \tag{2.43}$$

$$\cdot \left\{ \exp\left(i\Delta m_d t/2\right) \exp\left[-\left(\Delta\Gamma/2\right)t/2\right] \pm \exp\left(-i\Delta m_d t/2\right) \exp\left[+\left(\Delta\Gamma/2\right)t/2\right] \right\}$$
(2.44)

where m and Γ are the average mass and width of the two mass eigenstates, respectively, i.e. $m \equiv (m_{\rm H} + m_{\rm L})/2$ and $\Gamma \equiv (\Gamma_{\rm H} + \Gamma_{\rm L})/2$.

2.4.2 *B* Meson decays into *CP* eigenstates

In this section, we see the time evolution of neutral B meson decays into the final state f that is possible for both B^0 and \overline{B}^0 decays. We define the instantaneous decay amplitudes of B^0 and \overline{B}^0 to f and \overline{f} , which is the charge conjugation state of f, as:

$$A(f) \equiv \langle f | \mathcal{H}_{\text{int}} | B^0 \rangle, \qquad (2.45)$$

$$\overline{A}(f) \equiv \langle f | \mathcal{H}_{\text{int}} | \overline{B}^0 \rangle, \qquad (2.46)$$

$$A(\bar{f}) \equiv \langle \bar{f} | \mathcal{H}_{\text{int}} | B^0 \rangle \tag{2.47}$$

and



Figure 2.4: Diagrams for $B^0 - \overline{B}^0$ mixing.

$$\overline{A}(\overline{f}) \equiv \langle \overline{f} | \mathcal{H}_{\text{int}} | \overline{B}^0 \rangle, \qquad (2.48)$$

where \mathcal{H}_{int} is the Hamiltonian of weak interaction. A(f) and $\overline{A}(f)$ are equal to $A(\bar{f})$ and $\overline{A}(\bar{f})$, respectively, because $f = \bar{f}$. The time-dependent decay amplitude for a pure B^0 state at the time t = 0 to decay into a final state f at time t is obtained from Equations 2.40 and 2.41, and the instantaneous decay amplitudes as:

$$A(t; B^{0} \to f) = g_{+}(t)A(f) + \frac{q}{p}g_{-}(t)\overline{A}(f)$$

$$= A(f)\left[g_{+}(t) + \frac{q}{p}\overline{\rho}(f)g_{-}(t)\right], \qquad (2.49)$$

where $\overline{\rho}(f) \equiv \overline{A}(f)/A(f) = 1/\rho(f)$. The time-dependent decay amplitude for a pure \overline{B}^0 state at the time t = 0 to decay into a final state \overline{f} at time t is obtained with similar way as

$$A(t;\overline{B}^{0} \to \overline{f}) = g_{+}(t)\overline{A}(\overline{f}) + \frac{p}{q}g_{-}(t)A(\overline{f})$$

$$= \overline{A}(f)\left[g_{+}(t) + \frac{p}{q}\rho(f)g_{-}(t)\right].$$
(2.50)

The dime-dependent decay rates are thus given by

$$\Gamma(B^{0}(t) \to f) = |A(t; B^{0} \to f)|^{2}$$

$$= \frac{\exp(-\Gamma t)}{2} |A(f)|^{2} \left[\left(1 + |\lambda_{f}|^{2}\right) \cosh\left(\frac{\Delta\Gamma}{2}t\right) + \left(1 - |\lambda_{f}|^{2}\right) \cos(\Delta m_{d}t) -2\operatorname{Re}(\lambda_{f}) \sinh(\frac{\Delta\Gamma}{2}t) - 2\operatorname{Im}(\lambda_{f}) \sin(\Delta m_{d}t) \right]$$
(2.51)

and

$$\Gamma(\overline{B}^{0}(t) \to f) = |\overline{A}(t; \overline{B}^{0} \to f)|^{2}$$

$$= \frac{\exp(-\Gamma t)}{2} |\overline{A}(f)|^{2} \left[\left(1 + |\lambda_{f}|^{-2} \right) \cosh\left(\frac{\Delta\Gamma}{2}t\right) + \left(1 - |\lambda_{f}|^{-2} \right) \cos(\Delta m_{d}t) - 2\operatorname{Re}\left(\lambda_{f}^{-1}\right) \sinh\left(\frac{\Delta\Gamma}{2}t\right) - 2\operatorname{Im}\left(\lambda_{f}^{-1}\right) \sin(\Delta m_{d}t) \right], \quad (2.52)$$

where λ_f is defined as

$$\lambda_f \equiv \frac{q}{p} \cdot \overline{\rho}(f) = \frac{q}{p} \cdot \frac{1}{\rho(f)}.$$
(2.53)

In the B_d^0 meson system, $\Delta\Gamma/\Gamma$ is much smaller than unity, because the difference is produced by common decay channels to B^0 and \overline{B}^0 with branching fractions of 10^{-3} or less. Therefore, the time-dependent decay rates are approximated as

$$\Gamma(B^{0}(t) \to f) = \frac{\exp(-\Gamma t)}{2} |A(f)|^{2} \left[\left(1 + |\lambda_{f}|^{2} \right) + \left(1 - |\lambda_{f}|^{2} \right) \cos(\Delta m_{d} t) - 2 \operatorname{Im}(\lambda_{f}) \sin(\Delta m_{d} t) \right]$$
(2.54)

and

$$\Gamma(\overline{B}^{0}(t) \to f) = \frac{\exp(-\Gamma t)}{2} |\overline{A}(f)|^{2} \left[\left(1 + |\lambda_{f}|^{-2}\right) + \left(1 - |\lambda_{f}|^{-2}\right) \cos(\Delta m_{d} t) - 2\operatorname{Im}\left(\lambda_{f}^{-1}\right) \sin(\Delta m_{d} t) \right] 2.55 \right]$$

A time-dependent CP-violating asymmetry is defined as the normalized decay difference:

$$A_{CP}(t;f) \equiv \frac{\Gamma(\overline{B}^{0}(t) \to f) - \Gamma(B^{0}(t) \to f)}{\Gamma(\overline{B}^{0}(t) \to f) + \Gamma(B^{0}(t) \to f)}$$

$$= \frac{|\lambda_{f}|^{2} - 1}{|\lambda_{f}|^{2} + 1} \cos(\Delta m_{d}t) + \frac{2\operatorname{Im}(\lambda_{f})}{|\lambda_{f}|^{2} + 1} \sin(\Delta m_{d}t).$$
(2.56)

The *CP* violation that appears in cosine term comes from difference of decay amplitude between B^0 and \overline{B}^0 to same *CP* eigenstate. This kind of *CP* violation referred to as *direct CP violation*. On the other hand, the *CP* violation which appears in sin term is due to the interference between a decays with and without mixing which can occur whenever $\text{Im}(\lambda_f) \neq 0$. This kind of *CP* violation is called *mixing-induced CP violation*.

2.4.3 ϕ_1 measurement in $B^0 \to J \psi K_S$ decays

 $B^0 \to J\psi K_S$ decays, where K_S decays into two charged pions, is the promising decay mode to extract ϕ_1 experimentally. This decay mode is called *golden mode*, because $B^0 \to J\psi K_S$ decays can be reconstructed with a large S/N ratio. The diagrams of $B^0 \to J\psi K_S$ decays are shown in Figure 2.5(a) and 2.5(b), which are called *tree diagram* and *penguin diagram*, respectively. The amplitude of the tree diagram, $A_{\text{tree}}(J\psi K_S)$, and the penguin diagrams $A_{\text{penguin}}(J\psi K^0)$, are

$$A_{\text{tree}}(B^0 \to J\psi K^0) \equiv \langle J/\psi K^0 | \mathcal{H}_{\text{tree}} | B^0 \rangle = V_{cb}^* V_{cs} A_{\text{tree}}$$
(2.57)

and

$$A_{\text{penguin}}(B^0 \to J\psi K^0) \equiv \langle J/\psi K^0 | \mathcal{H}_{\text{penguin}} | B^0 \rangle = \sum_{q=u,c,t} V_{qb}^* V_{qs} A_{\text{penguin}}^q, \qquad (2.58)$$

where A_{tree} , and A_{penguin}^q are the amplitudes apart from the explicitly shown CKM matrix elements. Using the unitarity condition, $\sum_{q=u,c,t} V_{qb}^* V_{qs} = 0$, A_{penguin}^q is expressed as

$$A_{\text{penguin}}(J\psi K^{0}) = V_{ub}^{*}V_{us}(A_{\text{penguin}}^{u} - A_{\text{penguin}}^{t}) + V_{cb}^{*}V_{cs}(A_{\text{penguin}}^{c} - A_{\text{penguin}}^{t})$$

$$\simeq V_{cb}^{*}V_{cs}(A_{\text{penguin}}^{c} - A_{\text{penguin}}^{t}), \qquad (2.59)$$

because the magnitude of $(V_{ub}^* V_{us})/(V_{cb}^* V_{cs})$ is equal to $\mathcal{O}(\lambda^2)$ according to the Wolfenstein parameterization. Equation 2.59 indicates that the weak phase in the penguin diagrams is the same as in the tree diagram. Therefore, we can neglect the contribution from the weak phase in other than the tree diagram, and we can conclude

$$|A(B^0 \to J\psi K^0)| = |A(\overline{B}^0 \to J\psi \overline{K}^0)|$$
(2.60)

and

$$\frac{A(\overline{B}^0 \to J\psi\overline{K}^0)}{A(B^0 \to J\psi\overline{K}^0)} = \frac{V_{cb}V_{cs}^*}{V_{cb}^*V_{cs}}.$$
(2.61)

Then we consider the K^0 - \overline{K}^0 mixing because we have K_S in the final state. By similar discussion in Section 2.4.1,

$$\langle K_S | = p_K^* \langle K^0 | - q_K^* \langle \overline{K}^0 |, \qquad (2.62)$$

where the ratio of q_K to p_K is estimated by the similar way to extract Equation 2.32 using $|V_{cd}^*V_{cs}| \gg |V_{td}^*V_{ts}|$ as

$$\frac{q_K^*}{p_K^*} \simeq \frac{V_{cs}V_{cd}^*}{V_{cs}^*V_{cd}}.$$
(2.63)

The amplitude for $B^0 \to J/\psi K_S$ is obtained as

$$A(B^{0} \to J\psi K_{S}) \equiv \langle J/\psi K_{S} | B^{0} \rangle = \langle K_{S} | K^{0} \rangle \langle J/\psi K^{0} | B^{0} \rangle$$

$$= p_{k}^{*} A(B^{0} \to J\psi K^{0})$$
(2.64)

and

$$A(\overline{B}^{0} \to J\psi K_{S}) \equiv \langle J/\psi K_{S} | \overline{B}^{0} \rangle = \langle K_{S} | \overline{K}^{0} \rangle \langle J/\psi \overline{K}^{0} | \overline{B}^{0} \rangle$$

$$= q_{k}^{*} A(\overline{B}^{0} \to J\psi \overline{K}^{0}). \qquad (2.65)$$

Here, we use the facts $CP|B^0\rangle = -|\overline{B}^0\rangle$, $CP|K^0\rangle = -|\overline{K}^0\rangle$, $CP|J/\psi\rangle = |J/\psi\rangle$ derived from $J^{CP} = 1^{--}$ for J/ψ , and $CP|J/\psi K^0\rangle = |J/\psi K^0\rangle$ because the angular momentum of the system should be 1.

2.4. CP VIOLATION IN THE B MESON SYSTEM

Thus, λ_f (Equation 2.53) for $B^0 \to J/\psi K_S$ decays is obtained as

$$\lambda_{J/\psi K_S} = -\frac{A(\overline{B}^0 \to J\psi \overline{K}^0)}{A(B^0 \to J\psi \overline{K}^0)} \cdot \left(\frac{q}{p}\right) \cdot \left(\frac{q_K^*}{p_K^a st}\right)$$
(2.66)

$$\simeq -\frac{V_{cb}V_{cs}^*}{V_{cb}^*V_{cs}} \cdot \frac{V_{tb}^*V_{td}}{V_{tb}V_{td}^*} \cdot \frac{V_{cs}V_{cd}^*}{V_{cs}^*V_{cd}}$$
(2.67)

$$= -\exp\left(-2i\phi_1\right),\tag{2.68}$$

where the minus sign is caused that the state of $|J/\psi K_S\rangle$ is *CP* odd. As results, Equations 2.54, 2.55 and 2.56 for $B^0 \rightarrow J/\psi K_S$ decays become much simpler:

$$\Gamma(B^{0}(t) \to J/\psi K_{S}) = \exp(-\Gamma t) |A(J/\psi K_{S})|^{2} \left[1 - \sin(2\phi_{1})\sin(\Delta m_{d}t)\right], \qquad (2.69)$$

$$\Gamma(B^{0}(t) \to J/\psi K_{S}) = \exp(-\Gamma t) |A(J/\psi K_{S})|^{2} [1 + \sin(2\phi_{1})\sin(\Delta m_{d}t)], \qquad (2.70)$$

and

$$A_{CP}(t; J/\psi K_S) \equiv \sin(2\phi_1)\sin(\Delta m_d t).$$
(2.71)

According to Equation 2.71, ϕ_1 is obtained from the time-dependent *CP*-violating asymmetry for $B^0 \to J/\psi K_S$ directly.



(a) Tree Diagram.

(b) (Strong) Penguin diagram. In the case of electroweak penguin contributions, the gluons are replaced by a Z of a γ .

Figure 2.5: Diagrams for $B \to J/\psi K_S$ decays.

2.4.4 *CP* violation in $B^0 \to \pi^+\pi^-$ decays

The diagrams of $B^0 \to \pi^+\pi^-$ decays are shown in Figure 2.6. The amplitude for the tree diagram shown in Figure 2.6(a), is

$$A_{\text{tree}}(B^0 \to \pi^+ \pi^-) \equiv \langle \pi^+ \pi^- | \mathcal{H}_{\text{tree}} | B^0 \rangle = V_{ub}^* V_{ud} A_{\text{tree}}, \qquad (2.72)$$

where A_{tree} is the amplitude apart from the explicitly shown CKM matrix elements. On the other hand, the amplitude for the penguin diagrams shown in Figure 2.6(b), is expressed as

$$A_{\text{penguin}}(B^{0} \to \pi^{+}\pi^{-}) \equiv \langle \pi^{+}\pi^{-} | \mathcal{H}_{\text{penguin}} | B^{0} \rangle = \sum_{q=u,c,t} V_{qb}^{*} V_{qd} A_{\text{penguin}}^{q}$$
$$= V_{ub}^{*} V_{ud} (A_{\text{penguin}}^{u} - A_{\text{penguin}}^{t}) + V_{cb}^{*} V_{cd} (A_{\text{penguin}}^{c} - A_{\text{penguin}}^{t})$$
(2.73)

where $A_{\text{penguin}}^q(q = u, c, t)$ are the amplitudes apart from the explicitly shown CKM matrix elements, and we use the unitarity condition, $\sum_{q=u,c,t} V_{qb}^* V_{qd} = 0$. In this case, the contribution from the penguin diagrams is not negligible, because both $V_{ub}^* V_{ud}$ and $V_{tb}^* V_{td}$ are approximately the same order of $\mathcal{O}(\lambda^3)$ according to the Wolfenstein parameterization.

If we could ignore the penguin contribution and consider the tree diagram only, we got

$$(A(B^0 \to \pi^+ \pi^-))^* \equiv V_{ub}^* V_{ud} A_{\text{tree}}$$
 (2.74)

and

$$(2.75) \quad (2.75) \quad (2.75) \quad (2.75)$$

because the $\pi^+\pi^-$ state is CP even, then we obtained λ_f defined in Equation 2.53 for $B^0 \to \pi^+\pi^-$ decays as

$$\simeq -\frac{\mathbf{v}_{ub}\mathbf{v}_{ud}}{V_{ub}^*V_{ud}} \cdot \frac{\mathbf{v}_{tb}\mathbf{v}_{td}}{V_{tb}V_{td}^*} \tag{2.77}$$

$$= \exp\left(+2i\phi_2\right), \tag{2.78}$$

and the time-dependent decay rates and the CP violation asymmetry as

$$(\Gamma(B^{0}(t) \to \pi^{+}\pi^{-}))^{*} = \exp(-\Gamma t) |A(\pi^{+}\pi^{-})|^{2} [1 + \sin(2\phi_{2})\sin(\Delta m_{d}t)],$$

$$(2.79)$$

$$(\Gamma(B^{0}(t) \to \pi^{+}\pi^{-}))^{*} = \exp(-\Gamma t)|A(\pi^{+}\pi^{-})|^{2} \left[1 - \sin(2\phi_{2})\sin(\Delta m_{d}t)\right],$$
(2.80)

and

$${}^{"}A_{CP}(t;\pi^{+}\pi^{-})" = \sin(2\phi_{2})\sin(\Delta m_{d}t),$$

$$(2.81)$$

respectively. In actual case,

$$|A(B^0 \to \pi^+ \pi^-)| \neq |A(\overline{B}{}^0 \to \pi^+ \pi^-)|$$

$$(2.82)$$

and

$$\overline{\rho}(\pi^+\pi^-) \neq 1 \tag{2.83}$$

due to the concurrent of two facts:

- The final state, which happens to be CP even, is made up with a combination of the isospin states with of I = 1 and I = 2, which can conceivably posses significantly different strong phase shifts.
- As already described, the penguin contribution is not negligible (*penguin pollution*), which affects the final state with an isospin of I = 1 only.

The time-dependent CP-violating asymmetry in $B^0 \to \pi^+\pi^-$ decays is

$$A_{CP}(t;\pi^+\pi^-) = \mathcal{A}_{\pi\pi}\cos(\Delta m_d t) + \mathcal{S}_{\pi\pi}\sin(\Delta m_d t), \qquad (2.84)$$

where $\mathcal{A}_{\pi\pi}$ and $\mathcal{S}_{\pi\pi}$ are defined as

$$\mathcal{A}_{\pi\pi} = \frac{|\lambda_{\pi^+\pi^-}|^2 - 1}{|\lambda_{\pi^+\pi^-}|^2 + 1} \tag{2.85}$$

and

$$S_{\pi\pi} = \frac{2 \operatorname{Im}(\lambda_{\pi^+\pi^-})}{|\lambda_{\pi^+\pi^-}|^2 + 1},$$
(2.86)

respectively. Here $\lambda_{\pi^+\pi^-}$ is defined as

$$\lambda_{\pi^+\pi^-} \equiv \frac{q}{p} \cdot \frac{A(\overline{B}^0 \to \pi^+\pi^-)}{A(\overline{B}^0 \to \pi^+\pi^-)},\tag{2.87}$$

and $|\lambda_{\pi^+\pi^-}| = |A(\overline{B}{}^0 \to \pi^+\pi^-)/A(\overline{B}{}^0 \to \pi^+\pi^-)|.$

2.4.5 Extract ϕ_2 information from the *CP* violation parameters in $B^0 \to \pi^+\pi^-$ decays

In this section, we discuss the method to extract the ϕ_2 information from the observable parameters, $\mathcal{A}_{\pi\pi}$ and $\mathcal{S}_{\pi\pi}$ in the *CP*-violating asymmetry of $B^0 \to \pi^+\pi^-$ decays. M. Gronau et al. propose the method to subtract the penguin pollution using the branching fractions of $B^0 \to \pi^+\pi^-$, $B^+ \to \pi^+\pi^0$, $B^0 \to \pi^0\pi^0$ and their chargeconjugate decays [35–37]. However, the method is difficult because $B^0 \to \pi^0\pi^0$ decays are not observed yet. In this section, we discuss based on the model-dependent method [38,39]. The amplitude of $B^0 \to \pi^+\pi^-$ decay is obtained from Equations 2.72 and 2.73, and is parameterized as

$$A(B^{0} \to \pi^{+}\pi^{-}) = V_{ub}^{*}V_{ud}(A_{\text{tree}} + A_{\text{penguin}}^{u} - A_{\text{penguin}}^{\iota}) + V_{cb}^{*}V_{cd}(A_{\text{penguin}}^{c} - A_{\text{penguin}}^{\iota})$$

$$\equiv -(V_{cd}V_{cb}^{*}) \cdot [|T|\exp(i\delta_{T})\exp(i\phi_{3}) + |P|\exp(i\delta_{P})], \qquad (2.88)$$



(a) Tree

(b) (Strong) Penguin. In the case of electroweak penguin contributions, the gluon is replaced by Z or γ .

Figure 2.6: Diagrams for $B^0 \to \pi^+\pi^-$ decays.

where $|T|(\delta_T)$ and $|P|(\delta_P)$ are the amplitudes (strong phase) of the contributions with and without the weak phase of ϕ_3 , respectively. The former is dominated by the tree diagram, and the latter is dominated by the penguin diagrams. The amplitude of $\overline{B}^0 \to \pi^+\pi^-$ are parameterized with same way as

$$A(\overline{B}^{0} \to \pi^{+}\pi^{-}) = -(V_{cd}^{*}V_{cb}) \cdot [|T|\exp(i\delta_{T})\exp(-i\phi_{3}) + |P|\exp(i\delta_{P})].$$
(2.89)

Hence, $\lambda_{\pi^+\pi^-}$ is obtained as

$$\lambda_{\pi^+\pi^-} = \frac{V_{tb}^* V_{td}}{V_{tb} V_{td}^*} \cdot \frac{V_{cd}^* V_{cb}}{V_{cd} V_{cb}^*} \cdot \exp(-2i\phi_3) \frac{1 + |P/T| \exp[i(\delta + \phi_3)]}{1 + |P/T| \exp[i(\delta - \phi_3)]}$$
(2.90)

$$= \exp(2i\phi_2)\frac{1+|P/T|\exp[i(\delta+\phi_3)]}{1+|P/T|\exp[i(\delta-\phi_3)]},$$
(2.91)

where $\delta \equiv \delta_P - \delta_T$, and $\phi_2 = \pi - \phi_1 - \phi_3$ is assumed. Therefore, $\mathcal{A}_{\pi\pi}$ and $\mathcal{S}_{\pi\pi}$ are parameterized as

$$\mathcal{A}_{\pi\pi} = \frac{2|P/T|\sin(\phi_1 + \phi_2)\sin\delta}{\mathcal{R}_{\pi\pi}}$$
(2.92)

and

$$S_{\pi\pi} = \frac{\sin 2\phi_2 + 2|P/T|\sin(\phi_1 - \phi_2)\cos\delta - |P/T|^2\sin 2\phi_1}{\mathcal{R}_{\pi\pi}},$$
(2.93)

respectively, where

$$\mathcal{R}_{\pi\pi} = 1 - 2|P/T|\cos\delta\cos(\phi_1 + \phi_2) + |P/T|^2.$$
(2.94)

Here, we take $-180^{\circ} \le \delta \le 180^{\circ}$. When $\mathcal{A}_{\pi\pi}$ is positive and $0^{\circ} < \phi_1 + \phi_2 < 180^{\circ}$, δ is negative.

Because ϕ_1 can be determined using $B \to J/\psi K_S$ decays, if we know a value of |P/T|, we can extract ϕ_2 from $\mathcal{A}_{\pi\pi}$ and $\mathcal{S}_{\pi\pi}$. Recent theoretical estimates prefer $|P/T| \sim 0.3$ [38, 40, 41]. In this paper, we use |P/T| predicted by M. Gronau et al. [38]:

$$|P/T| = 0.28 \pm 0.06, \tag{2.95}$$

where |P| is estimated from the $B^+ \to K^0 \pi^+$ decay rate assuming the SU(3) flavor symmetry, and |T| is estimated from $B \to \pi \ell \nu$ decay rate with factorization [40].

2.4.6 *CP* violation in $B^0 \to K^+\pi^-$ decays

In this section, we discuss the possibility that $B^0 \to K^+\pi^-$ decays, which contribute to the background of $B^0 \to \pi^+\pi^-$ reconstruction, have a *CP*-violating asymmetry. Since the final state, $K^{\pm}\pi^{\mp}$, is not the *CP* eigenstate, the mixing-induced *CP* violation like in $B^0 \to J/\psi K^0$ decays does not exist in this decay channel. However, another sort of *CP* violation called direct *CP* violation can exist in $B^0 \to K^+\pi^-$ decays. Figure 2.7 shows the diagrams contributing to the $B^0 \to K^+\pi^-$ decays. The contributions from the tree diagrams are suppressed by CKM

matrix elements of $\mathcal{O}(\lambda^4)$ according to the Wolfenstein parameterization, while that from the penguin contribution is $\mathcal{O}(\lambda^2)$. The decay amplitudes of $B^0 \to K^+\pi^-$ and $\overline{B}^0 \to K^-\pi^+$ decays are written as

$$A(B^{0} \to K^{+}\pi^{-}) = a_{T} \exp[i(+\phi_{T} + \delta_{T})] + a_{P} \exp[i(+\phi_{P} + \delta_{P})]$$
(2.96)

$$A(\overline{B}^0 \to K^- \pi^+) = a_T \exp[i(-\phi_T + \delta_T)] + a_P \exp[i(-\phi_P + \delta_P)], \qquad (2.97)$$

where $a_T(a_P)$, $\phi_T(\phi_P)$ and $\delta_T(\delta_P)$ are the amplitude, the weak phase and the strong phase of the tree (penguin) diagram, respectively. Note that the weak phase changes the sign under the *CP* transformation, while the strong phase does not. The *CP*-violating asymmetry of the decay rate is obtained as

$$\mathcal{A}_{K\pi} \equiv \frac{|A(\overline{B}^{0} \to K^{-}\pi^{+})|^{2} - |A(B^{0} \to K^{+}\pi^{-})|^{2}}{|A(\overline{B}^{0} \to K^{-}\pi^{+})|^{2} + |A(B^{0} \to K^{+}\pi^{-})|^{2}} \\
= \frac{2|a_{T}||a_{P}|\sin\Delta\delta\sin\Delta\phi}{|a_{T}|^{2} + |a_{P}|^{2} + 2|a_{T}||a_{P}|\cos\Delta\delta\cos\Delta\phi},$$
(2.98)

where $\Delta \delta \equiv \delta_T - \delta_P$, and $\Delta \phi \equiv \phi_T - \phi_P$. Therefore, *CP*-violating asymmetry is produced, if both $\Delta \delta$ and $\Delta \phi$ have non-zero values. $\mathcal{O}(|a_T|) \simeq \mathcal{O}(|a_P|)$ is required for feasible *CP* asymmetry.

In the *CP*-violation measurement in $B^0 \to \pi^+\pi^-$ decays, sum of the time-dependent decay rates of $B^0 \to K^+\pi^$ and $\overline{B}{}^0 \to K^-\pi^+$ decays is the important quantities, because both $K^+\pi^-$ and $K^-\pi^+$ states can be recognized as the $\pi^+\pi^-$ state if the kaon mis-identified as a pion are.

The time-dependent decay rate for a pure B^0 state at the time t = 0 to decay into $K^+\pi^-$ or $K^-\pi^+$ is obtained using Equation 2.40 as:

$$\Gamma(t, K^{+}\pi^{-} + K^{-}\pi^{+}) = \frac{1}{2} \exp(-\Gamma t) \left[(1 + \cos(\Delta m_{d} t)) |A(B^{0} \to K^{+}\pi^{-})|^{2} + (1 - \cos(\Delta m_{d} t)) |A(\overline{B}^{0} \to K^{-}\pi^{+})|^{2} \right],$$
(2.99)

where the approximations of $\Delta\Gamma = 0$ and |q/p| = 1 are applied. The time-dependent decay rate for a pure \overline{B}^0 state at the time t = 0 to decay into $k^+\pi^-$ or $K^-\pi^+$ are also obtained as:

$$\overline{\Gamma}(t, K^{+}\pi^{-} + K^{-}\pi^{+}) = \frac{1}{2} \exp(-\Gamma t) \left[(1 + \cos(\Delta m_{d} t)) |A(\overline{B}^{0} \to K^{-}\pi^{+})|^{2} + (1 - \cos(\Delta m_{d} t)) |A(B^{0} \to K^{+}\pi^{-})|^{2} \right].$$
(2.100)

The time-dependent asymmetry of these decay rates is obtained as:

$$\mathcal{A}_{K\pi}(t) \equiv \frac{\overline{\Gamma}(t, K^{+}\pi^{-} + K^{-}\pi^{+}) - \Gamma(t, K^{+}\pi^{-} + K^{-}\pi^{+})}{\overline{\Gamma}(t, K^{+}\pi^{-} + K^{-}\pi^{+}) + \Gamma(t, K^{+}\pi^{-} + K^{-}\pi^{+})} \\
= \frac{(|A(\overline{B}^{0} \to K^{-}\pi^{+})|^{2} - |A(B^{0} \to K^{+}\pi^{-})|^{2})\cos(\Delta m_{d}t)}{|A(\overline{B}^{0} \to K^{-}\pi^{+})|^{2} + |A(B^{0} \to K^{+}\pi^{-})|^{2}} = \mathcal{A}_{K\pi}\cos(\Delta m_{d}t).$$
(2.101)

2.5 *CP* violation measurement in the asymmetric *B*-factory

In this section, we describe the concept of the measurement of the time-dependent CP-violating asymmetry in a B-factory experiment.

In the *B*-Factory, *B* mesons are produced from $b\bar{b}$ resonance state of $\Upsilon(4S)$, because the $\Upsilon(4S)$ is the lowest bound state that can be decay into a $B\bar{B}$ pair. $\Upsilon(4S)$ decays into a coherent $B^0-\bar{B}^0$ state with a *C*-odd configuration. Subsequently the time-evolution of the produced *B* mesons take place preserving the *C*-odd configuration. According to the Bose statistics, if one of the mesons is B^0 at a certain time, the other one cannot be B^0 then should be \overline{B}^0 , because the state must be odd under the exchange of two mesons. Let us consider the decay rate of *B* mesons in this system. We label each of the *B* mesons with its momentum. B_1 and B_2 have the momentum of k and -k, respectively. At the time t = 0, when $\Upsilon(4S)$ decays into *B* mesons pair, the state of B_1 - B_2 system is expressed as

$$B_1 B_2(t=0)\rangle = \frac{1}{\sqrt{2}} \left(\left| B_1^0 \right\rangle \left| \overline{B}_2^0 \right\rangle - \left| \overline{B}_1^0 \right\rangle \left| B_2^0 \right\rangle \right)$$

and



(a) Tree

(b) (Strong) Penguin. In the case of electroweak penguin contributions, the gluon is replaced by Z or γ .

Figure 2.7: Diagrams for $B^0 \to K^+\pi^-$ decays.

where B_1^0 (B_2^0) denotes B_1 (B_2) is B^0 and \overline{B}_1^0 (\overline{B}_2^0) denotes B_1 (B_2) is \overline{B}^0 , and the state is chosen to be antisymmetric because of the *C*-odd state of the system. Assuming $\Delta\Gamma = 0$, a state with B_1 at the time $t = t_1$ and B_2 at time $t = t_2$ can be expressed according to Equations 2.40 and 2.41 as

$$|B_1B_2(t_1,t_2)\rangle = \frac{1}{\sqrt{2}} \exp\left[-\frac{\Gamma}{2}(t_1+t_2)\right] \cdot \left[i\sin\left(\frac{\Delta m_d(t_2-t_1)}{2}\right) \cdot \left(\frac{p}{q}|B_1^0B_2^0\rangle - \frac{p}{q}|\overline{B}_1^0\overline{B}_2^0\rangle\right) + \cos\left(\frac{\Delta m_d(t_2-t_1)}{2}\right) \cdot \left(|B_1^0\overline{B}_2^0\rangle - |\overline{B}_1^0B_2^0\rangle\right)\right]$$

The decay rates are computed for the case in which one of the *B* meson, B_1 , decays into a *flavor-specific* state, X, such as the semi-leptonic decays, while the other one, B_2 , decays into a *flavor-nonspecific state*, f_{CP} , such as $J/\psi K_S$, $\pi^+\pi^-$, and so on: i.e. B^0 (\overline{B}^0) can decay into X (\overline{X}) while $B^0 \to \overline{X}$ ($\overline{B}^0 \to X$) decay is forbidden. $B^0 \to f_{CP}$ and $\overline{B}^0 \to f_{CP}$ are allowed. Using the definitions of $A(X) \equiv \langle X | B^0 \rangle = \langle X | \overline{B}^0 \rangle$, $A(f_{CP}) \equiv \langle f_{CP} | B^0 \rangle$, and $\lambda_f = (q/p) \cdot (\langle f_{CP} | \overline{B}^0 \rangle)$, the decay rates are calculated as

$$G_{-}(t_{1}, t_{2}) \equiv |\langle (X)_{1}(f_{CP})_{2} | B_{1}B_{2}(t_{1}, t_{2}) \rangle|^{2} = \frac{1}{4} \exp \left[-\Gamma(t_{1} + t_{2}) \right] |A(X)|^{2} |A(f_{CP})|^{2} \left\{ (1 + |\lambda_{f}|^{2}) + (1 - |\lambda_{f}|^{2}) \cos \left[\Delta m_{d}(t_{2} - t_{1}) \right] -2 \operatorname{Im}(\lambda_{f}) \sin \left[\Delta m_{d}(t_{2} - t_{1}) \right] \right\}$$

$$(2.102)$$

and

$$G_{+}(t_{1}, t_{2}) \equiv |\langle (X)_{1}(f_{CP})_{2} | B_{1}B_{2}(t_{1}, t_{2}) \rangle|^{2} = \frac{1}{4} \exp \left[-\Gamma(t_{1} + t_{2}) \right] |A(X)|^{2} |A(f_{CP})|^{2} |p/q|^{2} \left\{ (1 + |\lambda_{f}|^{2}) - (1 - |\lambda_{f}|^{2}) \cos \left[\Delta m_{d}(t_{2} - t_{1}) \right] \right.$$

$$\left. + 2 \operatorname{Im}(\lambda_{f}) \sin \left[\Delta m_{d}(t_{2} - t_{1}) \right] \right\}.$$

$$(2.103)$$

For the $f_{CP} = \pi^+ \pi^-$ case, the approximation of $|p/q| \simeq 1$ and the notations defined as Equations 2.85 and 2.86 simplify Equations 2.102 and 2.103:

$$G_{\pm}(t_1, t_2) \propto \exp[-\Gamma(t_1 + t_2)] \left\{ 1 \pm \mathcal{A}_{\pi\pi} \cos[\Delta m_d(t_2 - t_1)] \pm \mathcal{S}_{\pi\pi} \sin[\Delta m_d(t_2 - t_1)] \right\}.$$
 (2.104)

Let t_2 be the time when one of two *B*-meson decays into $\pi^+\pi^-$ state, which is the *CP* eigenstate, and t_1 be the time when another *B* meson decays into the flavor-specific state. We label the *B* meson decaying into the *CP* eigenstate as B_{CP} and the remaining *B* meson as B_{tag} , hereafter. A duration measurement from the B_{tag} decay to the B_{CP} decay, defined as $\Delta t \equiv t_2 - t_1$, provides $\mathcal{A}_{\pi\pi}$ and $\mathcal{S}_{\pi\pi}$ instead of the measurements of t_1 and t_2 as follows.

Because we care neither the individual decay time t_1 nor t_2 , we have to integrate Equation 2.104 with respect to t_1 and t_2 under the constraint $\Delta t = t_2 - t_1$. Using $t_1 \ge 0$ and $t_2 \ge 0$,

$$\iint dt_1 dt_2 G_{\pm}(t_1, t_2) \delta(t_2 - t_1 - \Delta t) \propto \exp(-\Gamma |\Delta t|) \left[1 \pm \mathcal{A}_{\pi\pi} \cos(\Delta m_d \Delta t) \pm \mathcal{S}_{\pi\pi} \sin(\Delta m_d \Delta t)\right]. \quad (2.105)$$

It is worth mentioning what happens if we swap B_1 and B_2 in Equations 2.102 and 2.103. In this case B_1 decays into a CP eigenstate and B_2 decays into a flavor specific state, and therefore the definition of Δt is flipped as $\Delta t = t_1 - t_2$. Thus, we get the exactly same function as Equation 2.105.

We obtain a Δt distribution function from Equation 2.105 as

$$f(\Delta t; q, \mathcal{A}_{\pi\pi}, \mathcal{S}_{\pi\pi}) = \frac{1}{4\tau_{B^0}} \exp\left(-\frac{|\Delta t|}{\tau_{B^0}}\right) \left\{1 + q \left[\mathcal{A}_{\pi\pi} \cos(\Delta m_d \Delta t) + \mathcal{S}_{\pi\pi} \sin(\Delta m_d \Delta t)\right]\right\}$$
(2.106)

where $\tau_{B^0} \equiv 1/\Gamma$ is a lifetime of B^0 meson and q = +1 (q = -1) for the case that B_{tag} is specified as B^0 (\overline{B}^0) . This function is normalized as

$$\sum_{q=\pm 1} \int \mathrm{d}(\Delta t) f(\Delta t; q, \mathcal{A}_{\pi\pi}, \mathcal{S}_{\pi\pi}) = 1.$$
(2.107)

Figure 2.9(a) shows the proper-time distributions for q = 1 and q = -1, respectively, where the both inputs of $\mathcal{A}_{\pi\pi}$ and $\mathcal{S}_{\pi\pi}$ are +0.7.

To summarize, in the $B^0\overline{B}^0$ system produced from the $\Upsilon(4S)$ decay, we can extract the parameters of *CP*-violating asymmetry by measuring the proper-time difference, Δt , and identifying the flavor of B_{tag} .

We describe the experimental procedure to determine the parameters of CP-violating asymmetry in an asymmetric *B*-factory. Figure 2.8 shows the conceptual drawing of the experimental procedure. In the *B*-factory, the accelerator produces a $B^0\overline{B}^0$ meson pair via the $\Upsilon(4S)$ resonance. One of the *B* meson, B_{CP} , is fully reconstructed using the flavor-nonspecific decay mode, such as $B^0 \to \pi^+\pi^-$, and its decay position is reconstructed using the daughter tracks. The decay position of B_{tag} is obtained from remaining tracks.

Because $\Upsilon(4S)$ is boosted in the laboratory frame in the asymmetric beam-energy collision, and the B^0 mesons pair is produced almost at rest in the $\Upsilon(4S)$ rest frame, the proper-time difference is obtained as

$$\Delta t = \frac{\Delta z}{c \cdot (\beta \gamma)_{\Upsilon(4S)}}, \qquad (2.108)$$

where Δz is the distance of decay positions of B_{CP} and B_{tag} in beam direction and $(\beta\gamma)_{\Upsilon(4S)}$ is the Lorentz boost factor of $\Upsilon(4S)$, which is obtained from the accelerator parameters. Since *B* meson lifetimes is about 1.5 ps and $(\beta\gamma)_{\Upsilon(4S)}$ is set to be about 0.5, *B* mesons run about 200 μ m before their decay, which is sizable length to be measured by the silicon strip detector. Because of the finite resolution of Δz measurement, the *CP*-violating asymmetry in Δt distribution is smeared, as shown in Figure 2.9(b). Thus, an appropriate understanding of the resolution is one of the essential components to measure the parameters of *CP*-violating asymmetry.

It is necessary to determine the flavor of B_{tag} . This process is called *flavor tagging*. The presence of the following particles can be used to tag the flavor of the *B* mesons: leptons from $b \to c\ell^-\bar{\nu}$, leptons from cascade decays $b \to c \to \ell^+ s\nu$, fast pions which reflects the charge of virtual *W* in $b \to c W$, slow pions from $D^{*\pm}$ whose charge reflects a charge of *c*, and Kaons and Λ from cascade decays of $b \to c \to s$. Using the probability of the incorrectly assignment of the B_{tag} flavor, w_{tag} (called *wrong tag fraction*), the observed Δt distribution becomes

$$f(\Delta t; q, \mathcal{A}_{\pi\pi}, \mathcal{S}_{\pi\pi}) = \frac{1}{4\tau_{B^0}} \exp\left(-\frac{|\Delta t|}{\tau_{B^0}}\right) \left\{1 + q(1 - w_{\text{tag}}) \left[\mathcal{A}_{\pi\pi} \cos(\Delta m_d \Delta t) + \mathcal{S}_{\pi\pi} \sin(\Delta m_d \Delta t)\right]\right\} \\ + \frac{1}{4\tau_{B^0}} \exp\left(-\frac{|\Delta t|}{\tau_{B^0}}\right) \left\{1 - qw_{\text{tag}} \left[\mathcal{A}_{\pi\pi} \cos(\Delta m_d \Delta t) + \mathcal{S}_{\pi\pi} \sin(\Delta m_d \Delta t)\right]\right\} \\ = \frac{1}{4\tau_{B^0}} \exp\left(-\frac{|\Delta t|}{\tau_{B^0}}\right) \left\{1 + q(1 - 2w_{\text{tag}}) \left[\mathcal{A}_{\pi\pi} \cos(\Delta m_d \Delta t) + \mathcal{S}_{\pi\pi} \sin(\Delta m_d \Delta t)\right]\right\} (2.109)$$

as shown in Figure 2.9(c). Thus, it is important to estimate w_{tag} correctly for the determination of the parameters of CP-violating asymmetry.

Figure 2.9(d) shows the expected Δt distributions for the input values of $\mathcal{A}_{\pi\pi} = +0.7$ and $\mathcal{S}_{\pi\pi} = +0.7$ where the detector resolution and the wrong tag fraction are realistically taken into accounted. We determine the parameters of CP-violating asymmetry from the asymmetric Δt distribution by the unbinned-maximum-likelihood-fit method.

2.6 Experimental Constraints on the Unitarity Triangle

Here we review the current experimental constraints on the unitarity triangle. Those measurements define the preferable area for ϕ_2 by specifying the apex of $(\bar{\rho}, \bar{\eta})$ in Figure 2.3(b).

The elements in the first row of the CKM matrix are accessible in the so-called direct (tree-level) processes, i.e. in weak decays of hadrons containing the corresponding quarks. $|V_{ud}|$ and $|V_{us}|$ are known with accuracies of



Figure 2.8: Conceptual drawing of the measurement of the proper-time difference for $B^0\overline{B}^0$ mason pairs in the asymmetric *B*-factory. In this illustration, B_{tag} and B_{CP} are assumed to be B^0 and \overline{B}^0 , respectively, at the decay time of B_{tag} .



Figure 2.9: Proper-time difference distributions for $B^0 \to \pi^+\pi^-$ decays with the *CP*-violation parameters of $\mathcal{A}_{\pi\pi} = +0.7$ and $\mathcal{S}_{\pi\pi} = +0.7$ for events with B_{tag} tagged as B^0 (solid lines) and tagged as \overline{B}^0 (dashed doted lines). The sum and average of them are indicated by dashed lines and dotted lines, respectively.

better than 1%. $|V_{cb}|$ is determined with an accuracy of 5%, and $|V_{cd}|$ and $|V_{cs}|$ are known with about 10-20% errors. Hence, the λ and A in Wolfenstein parameterization are rather well determined experimentally [42]:

$$\lambda = |V_{us}| = 0.2196 \pm 0.0026 \tag{2.110}$$

and

$$A = \frac{|V_{cb}|}{|V_{us}|^2} = 0.85 \pm 0.04, \tag{2.111}$$

respectively. On the other hand, $|V_{ub}|$ has an uncertainty of about 30%. $|V_{td}|$, which is determined from the $B^0 - \overline{B}^0$ mixing, also has the large uncertainty. This implies the existence of rather significant uncertainty in ρ and η .

To determine the shape of the triangle, one can aim for measurements of the two sides and three angles. Using Wolfenstein parameterization and Equations 2.23 and 2.24, the two side of the unitarity triangle are expressed as

$$R_b \equiv \sqrt{\bar{\rho}^2 + \bar{\eta}^2} = \frac{1}{\lambda} \left(1 - \frac{\lambda^2}{2} \right) \frac{|V_{ub}|}{|V_{cb}|}$$
(2.112)

and

$$R_t \equiv \sqrt{(1-\bar{\rho})^2 + \bar{\eta}^2} \simeq \frac{1}{A\lambda^3} |V_{td}V_{tb}^*|.$$
(2.113)

 $|V_{td}V_{tb}^*|$ is accessible through $B^0-\overline{B}^0$ mixing by the measurement of mass difference, Δm_d , assuming that the dominant contribution to the mass difference arises from the matrix element between a B^0 and \overline{B}^0 of an operator that corresponds to a box diagram, with W bosons and top quarks as sides. The theoretical prediction by the standard model is

$$|V_{td}V_{tb}^*| \left(\propto \sqrt{\Delta m_d}\right) = 0.0079 \pm 0.0015,$$
 (2.114)

where the uncertainty comes primary from that in the hadronic matrix elements. In the ratio of B_s to B_d mass difference, many common factors (such as the QCD correction and dependence on the top quark mass) cancel and gives another determination of R_t through a measurement of $B_s^0 - \overline{B}_s^0$ mixing,

$$R_t^2 = f \frac{m_{B_s}}{m_{B_d}} \frac{\Delta m_s}{\Delta m_d} \frac{1 - \lambda^2 (1 - 2\bar{\rho})}{\lambda^2}, \qquad (2.115)$$

where f is the ratio of the hadronic matrix element for B_s to B_d . Presently, only a lower limit on Δm_s is obtained, and thus it gives upper limit of R_t .

Another constraint is given by $K^0 - \overline{K}{}^0$ mixing parameter, ϵ_K . The constraint arising in the $\bar{\rho} - \bar{\eta}$ plane forms hyperbola, depending on the hadronic matrix elements.

 ϕ_1 is directly measured using the time-dependent CP asymmetry in $b \to c\bar{c}s$ decays. The present results from *B*-factory experiments, Belle [8] and *BABAR* [9], when averaged yield

$$\sin 2\phi_1 = 0.78 \pm 0.08. \tag{2.116}$$

Figure 2.10 shows the above constraints on the position of apex, $(\bar{\rho}, \bar{\eta})$, of the unitarity triangle.



Figure 2.10: Constraint on the position of apex of the unitarity triangle, $(\bar{\rho}, \bar{\eta})$. A possible unitarity triangle is shown with the apex in the preferred region.

Chapter 3

Experimental apparatus

In this chapter, the experimental apparatus used in the analysis is described. The analysis is based on the *B*-factory experiment at the *High Energy Accelerator Research Organization* (KEK), in Japan employing the KEKB electron-positron accelerator and the Belle detector.

KEKB produce a huge amount of B mesons with the highest luminosity in the world as of July 2002. We describe the detail of KEKB in Section 3.1. The Belle detector, which is the multipurpose particle detector complex attached to KEKB, described in Section 3.2. In the section, the trigger system to operate the detector, the data acquisition system to record the data and the offline computing facility to analyze the recorded data are also described in detail.

3.1 The KEKB accelerator

KEKB [43] is an asymmetric electron-positron collider designed to produce a large number of B mesons. The energy of electrons and positrons are 8.0 GeV and 3.5 GeV, respectively. The energy in the center-of-mass system (cms) is 10.58 GeV, which corresponds to the $\Upsilon(4S)$ resonance. The produced $\Upsilon(4S)$ is in motion with Lorentz boost factor, $(\beta\gamma)_{\Upsilon(4S)}$ equal to 0.425 and decays into neutral or charged B mesons pair. The flight length of Bmesons in laboratory frame is about 200 μ m, because their lifetimes are about 1.6 ps. The design luminosity of KEKB is 10^{34} cm⁻²s⁻¹, which corresponds to about $10^8 \Upsilon(4S)$ s a year.

The configuration of the KEKB accelerator is illustrated in Figure 3.1. KEKB consists of two storage rings, the *High Energy Ring* (HER) for electrons and the *Low Energy Ring* (LER) for positrons. HER and LER are about 3 km long in circumference. Electrons are generated by an electron gun and are then accelerated up to 8.0 GeV in an injector linear accelerator (*linac*) that is about 600 m long over all. Then the electron beam is directory injected into HER. Electrons are also injected to a tungsten target at the intermediate position of linac, and electron-positron pairs are produced there. Positrons are separated and accelerated up 3.5 GeV to in the rest path of the linac. Then positron beam is injected into LER. HER and LER cross at the one point, called the *interaction point* (IP). Electrons and positrons collide at IP with a finite angle of ± 11 mrad. The finite angle crossing minimizes the bending of beam and reduces the synchrotron radiation to the Belle detector.

KEKB construction was completed in November 1998 and commissioning started in December 1998. In July 2002, KEKB archived the peak luminosity of 7.348×10^{33} cm⁻²s⁻¹ with 1365 mA LER and 918 mA HER beam current, where the design values are 2.6 A for LER and 1.1 A for HER. KEKB delivered an integrated luminosity of 89.62 fb⁻¹ to Belle by July 2002.

3.2 The Belle Detector

The Belle detector [44, 45] is designed to measure the *B*-meson decay vertices with a sufficient resolution for the measurement of the time-dependent decay ratio, and to identify particles for the flavor determination of *B* mesons. The Belle detector consists of the several components as shown in Figure 3.2. The acceptance region is asymmetric because of the asymmetric beam energies.

Charged-particle reconstruction is provided by a wire drift chamber, called *Central Drift Chamber* (CDC) [46, 47], operated under a 1.5 T super-conducting solenoid. B meson decay points are measured by a *Silicon Vertex Detector* (SVD) [48,49] that is just outside the beryllium beam pipe. The charged-particle identification is provided by the dE/dx measurement in CDC, and information from the *Aerogel Čerenkov Counter* (ACC) [50,51] and the



Figure 3.1: The KEKB accelerator

Time of flight counter (TOF) [52]. The set of trigger modules, Thin Trigger Scintillation Counter (TSC), is attached to TOF. Photon detection and Electron identification are carried out using the Electromagnetic Calorimeter (ECL) [53–55] made of thallium doped CsI crystals. Resistive plate counters for K_L meson detection and muon identification (KLM) [56] are interspersed within the magnet-flux return iron of the solenoid that also works as an absorber. Particles moving very close to the beam direction are detected by the BGO Extreme Forward Calorimeter (EFC) [57,58] placed on the surface of the cryostats for final focusing magnets in the forward and backward regions. The characteristics and performance of each subcomponents is summarized are Table 3.1.

The Belle detector construction was completed in December 1998. After calibration with 100K cosmic ray events, it was rolled into the interaction point and started recording the e^+e^- collision data in May 1999.

The coordinate system of the Belle detector is defined as follows:

- x: horizontal outward to the KEKB ring,
- y : vertical upward,
- z : opposite of the positron beam direction,
- r : $\sqrt{x^2 + y^2}$,
- θ : polar angle measured from +z direction,
- ϕ : azimuthal angle around z axis.

3.2.1 Beam Pipe

To achieve the precise determination of decay points in SVD, it is required to minimize the thickness of the beam pipe at the interaction region, because the multiple Coulomb scattering at the material inside the first layer of SVD is the limiting factor on the decay vertex resolution. Therefore, the beam pipe wall is made of beryllium.

Because the beam-induced heating reaches a few hundred watts, the beam pipe is also required to have an active cooling system and a mechanism for shielding the SVD from the heat. The structure of the beam pipe is shown in Figure 3.3. The beam pipe consists of two cylinders with different radii, r = 20.0 mm and 23.0 mm. The thickness of each cylinder is 0.5 mm. The gap within two cylinders is filled with helium gas for cooling.

3.2.2 Silicon Vertex Detector

The main task of Silicon Vertex Detector (SVD) is to reconstruct the decay points of two primary B masons to observe the time-dependent CP asymmetries. It requires the measurement of the distance of two decay vertices



Figure 3.2: Overview of the Belle detector.



Figure 3.3: The structure of the beam pipe at interaction point.

Component	Type	Configuration
Component	Readout channel	Performance/Note
Beam Pipe	Beryllium double-wall	Inner radius = 20 mm, Thickness = $0.5(Be)/2.5(He)/0.5(Be)$ mm
		Helium gas chilled
SVD	Double sided silicon strip	300 μ m thickness, 3 layers: $r = 30.0 \sim 60.5$ mm, Strip pitch: $25(p)/50(n) \ \mu$ m
572	40.96K $ch(\theta)$, 40.96K $ch(r-\phi)$	Δz resolution ~ 200 μm in R.M.S.
CDC	Small-cell Drift chamber	Anode: 50 layers, Cathode: 3 layers, $r = 83 \sim 863 \text{ mm}, z = -77 \sim 160 \text{ cm}$
CDC	8.4K ch(Anode), 1.8K ch(Cathode)	$\begin{aligned} \sigma_{r \to \phi} &= 130 \; \mu \text{m}, \; \sigma_z = 200 \sim 1400 \; \mu \text{m}, \\ \sigma_{p_t} / p_t &= (0.29 \oplus 0.20 \cdot p_t [\text{GeV}/c])\%, \; \sigma_{\text{d}E/\text{d}x} = 7\% \end{aligned}$
ACC	Silica aerogel Čerenkov counter	$\sim 120 \times 120 \times 120 \text{ mm}^3$ block: 960 barrel and 228 forward end-cap, Fine Mesh PMT readout
	1788 ch	π^{\pm}/K^{\pm} separation: 1.2
TSC	Scintillation counter	64 ϕ segments, $r=1175$ mm, 3 m long, attached to TOF
	64 ch	
TOF	Scintillation counter	128 ϕ segments, $r=1201$ mm, 3 m long
	128×2 ch	$\sigma_t = 100 \text{ ps}, \pi^{\pm}/K^{\pm} \text{ separation: } p < 1.2 \text{ GeV}/c$
	CsI(Tl)	Towered structure, $\sim 55 \times 55 \times 300 \text{ mm}^3$ block, $r = 125 \sim 162 \text{ cm(barrel)}, z = -102, 196 \text{ cm(endcap)}$
ECL	6624 ch(Barrel), 1152 ch(Forward), 960 ch(Backward)	$\sigma_E/E = (1.34 \oplus 0.066 \cdot E^{-1} \oplus 0.81 \cdot E^{-1/4})\%,$ $\sigma_{\text{position}} = 0.27 + 3.4 \cdot E^{-1/2} + 1.8 \cdot E^{-1/4} \text{ mm (E in GeV)}$
Magnet	Super-conducting solenoid	Inner radius $= 1,700 \text{ mm}$
•		B = 1.5 T
KI M	Glass resistive plate counters	14 layers: 50 mm Fe + 40 mm gap, 2 RPCs(θ strips and ϕ strips) in each gap
IXLIVI	$\begin{array}{c} 16 \mathrm{K} \operatorname{ch}(\theta), \\ 16 \mathrm{K} \operatorname{ch}(r) \end{array}$	$\Delta \theta = \Delta \phi = 30 \text{ mrad for } K_L, \sigma_t = \text{a few ps}$
	$\mathrm{Bi}_4\mathrm{Ge}_3\mathrm{O}_{12}$	32 segments in ϕ , 5 segments in θ , Photodiode readout
EFC	5 ch(θ), 32 ch(ϕ)	Energy Resolution (R.M.S.) = $7.3\%(8 \text{ GeV})$, = $5.8\%(3.5 \text{ GeV})$

Table 3.1: Summary of the subcomponents of the Belle detector.

along z-axis with a good precision comparing to the B meson lifetimes: ~ 200 μ m. In addition, SVD is useful to identify and to measure the decay vertices of D and τ particles.

SVD consists of three layers of *Double sided Silicon Strip Detector* (DSSD) surrounding the beam pipe. SVD has 102 DSSDs in total, having a total 81,920 readout channels. Figure 3.4 shows the structure of SVD. The radial positions of layers are at r = 300, 455 and 605 mm, which have 8, 10, 14 sensor modules (*full-ladder*) in ϕ direction, respectively. The polar-angle coverage is $23^{\circ} < \theta < 139^{\circ}$ corresponding to 86% of the full solid angle. The each full-ladder consists of two *half-ladders* and a support rib. The half-ladders consist of DSSDs, ceramic-hybrid preamplifier circuit cards, heat sink made of boron-nitride and copper heat pipe filled with water vapor. Two kinds of half-ladders, *short* and *long*, are used to construct the full-ladder. A short half-ladder has a single DSSD, while a long half-ladder has two DSSDs where strips on the *n*-side (*p*-side) of one DSSD are wire-bonded to strips on the *p*-side (*n*-side) of the other DSSDs to keep a good *signal-to-noise*(s/N) ratio. Full-ladder and one long half-ladder. Full-ladders in outermost layer consist of two short half-ladders, while those in middle layer consist of one short half-ladder and one long half-ladder. Full-ladders in outermost layer consist of two long half-ladders. The support ribs are made of boron-nitride reinforced by CFRP (*Carbon Fiber Reinforced Plastics*). The total amount of material is 0.5 % radiation length per layer.

The DSSDs were fabricated by Hamamatsu Photonics (HPK S6936) and were originally designed for the DELPHI microvertex detector [59]. The sensor size is $57.5 \times 33.5 \ mm^2$ and the thickness is 300μ m. Figure 3.5(a) shows the schematic structure of DSSD. The one side (n-side) of DSSD has n^+ -strips oriented perpendicular to the beam direction to measure z coordinate of tracks and the other side (p-side) has the p^+ -strip along the beam direction in order to measure the position of the track in r- ϕ plane. The strip pitches of p-side and n-side are $25 \ \mu\text{m}$ and $42 \ \mu\text{m}$, respectively. The bias voltage of 80 V is supplied to the n-side, and then the p side is grounded. The n^+ -strips are interleaved by the p^+ implants (p-stop) in order to separate the consecutive strips electrically. Figure 3.5(b) shows the readout scheme of DSSD. On the p-side, every second strips are connected to one readout channel, an aluminum electrode with AC coupling using high-resistive poly silicon. The remaining half strips are floating, which are biased but not connected to the preamplifiers, and are capacitively dividing the adjacent channels that are connected to the preamplifiers. On the n-side, adjacent strips are connected to one readout channel with AC coupling which gives an effective strip pitch of 84 μ m. In order to make the same direction of readout channels of the n^+ -side strips as that of the p-side, *Double Metal Layer* (DML) structure is adopted.

Each hybrid card has five VA1 chips [60], which are 128 ch amplifier LSI chips developed at CERN, to readout the signals from DSSDs. The VA1 chip has an excellent noise performance of $200e^- + 8e^-/pF$ in equivalent noise charge at a shaping time of 1.0 μ s [61]. The VA1 chip has also reasonably good radiation tolerance up to a dose of 200 krad. In summer 200, the original VA1s with 1.2- μ m technology were replaced with new VA1s processed with 0.8- μ m technology that has improved radiation tolerance up to a dose of 1 Mrad [62]. The multiplexed signal outputs from VA1s are transferred to *Fast Analog-to-Digital Converters* (FADCs) located about 30 m away from SVD by repeater modules located at the endcap regions of the Belle detector [63]. The transferred signals are digitized by FADCs, then the common-mode noise subtraction, data sparsification, and data formatting are performed online by *Digital Signal Processors* (DSPs) in the FADC modules.

The electrical noise of the half ladders are measured to be ~ 400 e^- , ~ 1000 e^- and ~ 1100 e^- for the *p*-side channels in short half ladders, and the channels in long half ladders, respectively. The S/N ratio for *Minimum-Ionizing Particles* (MIPs) is greater than 17. The intrinsic position resolution of DSSD, which is evaluated by the distance between the SVD hits in the middle layer and tracks reconstructed from SVD hits in the innermost and the outermost layers using the curvature information from CDC, is ~ 10 μ m for *p*-side and ~ 15 μ m for *n*-side [64]. The probability that a track reconstructed by CDC within the SVD acceptance ¹ has associated with SVD hits in at least two layers ² is measured to be higher than 97%. The resolution of impact parameter to the interaction point for tracks is measured using the tracks from $\gamma\gamma \rightarrow e^+e^-$, $\gamma\gamma \rightarrow 2\rho^0 \rightarrow 2(\pi^+\pi^-)$ and $e^+e^- \rightarrow \mu^+\mu^-$ events in beam collision and the cosmic ray events. The obtained impact parameter resolution, that depends on the momentum and polar angular of tracks, is shown in Figure 3.6, and well represented by the following formula: $(19 \oplus 50/p\beta \sin^{2/3} \theta) \ \mu$ m in *r*- ϕ plane and $(36 \oplus 42/p\beta \sin^{2/5} \theta) \ \mu$ m in *z* direction³.

3.2.3 Central Drift Chamber

The task of Central Drift Chamber (CDC) is the detection of charged particles and the measurement of the momentum of the tracks determined from the curvature of the charged tracks in the magnetic field of 1.5 T. CDC also measures the *energy loss* (dE/dx), and this dE/dx information is useful for the particle identification in the

¹Tracks from $K_S \to \pi^+\pi^-$ decays are excluded because K_S can be out side of innermost layer of SVD.

²Furthermore, it is also required at least one SVD hits to have both z and r- ϕ information.

³The unit of the momentum is GeV/c



Figure 3.4: Configuration of SVD



(a) Conceptual drawing of DSSD

(b) Readout scheme of DSSD

Figure 3.5: The schematic view of the double sided silicon strip detector



Figure 3.6: The impact parameter resolution of SVD.

momentum region of p < 0.8 GeV/c and p > 2 GeV/c, because the amount of dE/dx depends on the $\beta = v/c$ according to Bethe-Bloch formula.

Figure 3.7 shows the structure of CDC. The inner and the outer radii are 83 and 874 mm, respectively, and the polar-angle coverage is $15^{\circ} < \theta < 150^{\circ}$. CDC is a cylindrical wire drift chamber that contains 50 layers of anode wire and three cathode strip layers. The anode layers consist of 32 axial-wire layers and 18 stereo-wire layers. The axial wires are configured to be parallel to z-axis, while the stereo wires are slanted approximately ± 50 mrad, and provide the z coordinate measurements of tracks. Thus, CDC reconstructs the 3-dimensional particle trajectories. The chamber has a total 8,400 drift cells, which have are the rectangle structure of $16 \sim 20(\phi) \times 15 \sim 17(r)\mu m^2$, and each cell consists of 1 sense wire surrounded by 8 field wires. The sense wires are made of gold plated tungsten of $30 \ \mu m$ in diameter. The field wires are made of aluminum of $126 \ \mu m$ in diameter. The total numbers of the sense wires are 5,280 and 3,120 for axial and stereo, respectively, while the total number of the field wires is 33,344. The high voltage of 2.35 kV (typical) is supplied to the sense wires while the electric field on the surface of the field wires is kept below 20 kV/cm. A mixture of He (50%) and C_2H_6 (50%), which has a ~ 640 m radiation length, is filled in the chamber. The use of the helium minimizes the multiple Coulomb-scattering contribution to the momentum resolution. The drift velocity is about 4 cm/ μ m. A charged particle traversing in the chamber produces ionized gas of $\sim 100 \text{ cm}^{-1}$. A charge avalanche is caused by the ionized gas and it drifts to the sense wire with a finite drift time, then a signal is read out. At the inner most radius, three cathode strip layers are installed to provide the z position measurements of tracks for the trigger system [65, 66].

The charged-particle tracking is performed in three steps. At first, the track finding is performed by recognizing hit patterns. High transverse-momentum tracks are found in r- ϕ plane using axial wire hits, and then reconstructed 3-dimensionally by associating with stereo wire hits. Low transverse-momentum tracks are found with axial wire hits, stereo wire hits and SVD hit pattern information. At second stage, the trajectory of charged tracks are calculated using the Kalman Filter technique [67, 68] taking into account a non-uniformity of magnetic field, dE/dx and multiple scattering. SVD hits are associated with the reconstructed tracks with the same technique, and the trajectory is recalculated using CDC and SVD information to improve the momentum resolution. At last, the redundant tracks are removed by comparing track parameters each others.

The performance of CDC is evaluated with cosmic-ray tracks that pass thorough the interaction region.⁴ The spatial resolution of CDC is measured to be ~ 130 μ m. Figure 3.8(a) shows the spatial resolution as a function of the drift distance. The resolution of transverse momentum is measured as shown in Figure 3.8(b). It is parameterized as 0.20 $\cdot p_t \oplus 0.29/\beta$ % where p_t is the transverse momentum measured in GeV/c. The resolution of p_t is also measured using $e^+e^- \rightarrow \mu^+\mu^-$ process as 1.64 ± 0.04 % in the p_t range from 4.0 to 5.2 GeV/c.

The truncated-mean method is employed to estimate the most probable energy loss. The largest 20% of measured dE/dx value for each track is excluded, and the dE/dx value for each track is obtained by taking average of the remaining 80 % data, to avoid occasional large fluctuations in Landau tail of the dE/dx distributions.

 $^{^{4}}$ The cosmic rays are recognized by the tracking system as two tracks coming from the interaction region.

Figure 3.9(a) is a scatter plot of the measured dE/dx and the particle momentum for tracks in the beam collision data. Population of pions, kaons, protons and electrons can be clearly seen. The K/π separation more than 3σ is provided in the momentum range up to 0.8 GeV/c and above 2GeV/c. The e/π separation greater than 3σ is also provided in the momentum range from 0.3 to 3GeV/c. The dE/dx resolution for pions in the momentum range from 0.4 to 0.6GeV/c are measured to be 7.6% using $K_S \to \pi^+\pi^-$ decays as shown Figure 3.9(b), while the dE/dx resolution for electrons and muons are measured to be ~6% using Bhabha and $e^+e^- \to \mu^+\mu^-$ process, respectively.



Figure 3.7: Overview of the CDC structure. The lengths in the figure are in units of mm.



(a) Spatial resolution as a function of the drift distance.

(b) Transverse momentum resolution as the function of itself. The solid line shows the fitted result and the broken line represents an ideal expectation for $\beta = 1$ particles.

Figure 3.8: Performance of the CDC.

3.2.4 Aerogel Čerenkov Counter

The Aerogel Čerenkov Counter (ACC) provides the fine particle identification in the momentum range from 1.2 to 3.5 GeV/c. The ability of the good π/K separation in this momentum range is crucial to improve the S/N



(a) Measured dE/dx vs. momentum in beam collision data. The expected mean energy loss for pions, kaons, protons and electrons are also shown.

(b) Distribution for measured dE/dx divideed by expected value for pions from K_S decays.

Figure 3.9: Performance of dE/dx measurement in CDC.

ratio for the reconstruction of $B^0 \to \pi^+\pi^-$ Decays. When a particle passes thorough the silica aerogel radiator, the Čerenkov light is emitted if the following condition is satisfied:

$$m$$

where m the p are particle mass and momentum, and n is a refractive index of the radiator. By tuning the refractive index for the target particle momentum range, light particles such as pions emit the Cerenkov light in ACC, while heavier particles such as kaons do not, which is the basic concept of the particle identification by ACC. Silica aerogel used as a radiator is a low density and has a low refractive index between compressed gas and liquid materials, because it is a colloidal form of $(SiO_2)_n$ with more than 95% porosity. Figure 3.10 shows the side view of ACC. The barrel part consists of 960 counter modules, segmented into 60 cells in the ϕ direction. The forward endcap part consists of 228 counter modules, arranged in five concentric layers. The refractive index of aerogel ranges 1.010-1.028 for the barrel part depending on the polar angular regions, because the particles passing thorough the lower polar angle region has the large momentum. The refractive index in the endcap part is set to be 1.030 to provide the good π/K separation in the momentum range below 2.0 GeV/c for flavor tagging, because it is hard to produce the aerogel with the lower refractive index than 1.010. Figure 3.11 shows the structure of typical ACC modules for the barrel and the endcap parts. Because the ACC is operated under a magnetic field of 1.5 T, Fine-Mesh Photo-Multiplier Tubes (FM-PMTs) [69], with diameter of either 2, 2.5 or 3 in., are used to detect Cerenkov light. One or two FM-PMTs are attached to one counter module. The numbers of readout channels are 1560 in the barrel part and 228 in the endcap part. The performance of the counter module are measured using π^- and proton beams at KEK Proton Synchrotron [70] as shown in Figure 3.12(a). Figure 3.12(b) shows the measured pulse-height distribution in the barrel part for electrons from Bhabha processes and kaon candidates in hadronic events, which are identified by using information from TOF and dE/dx measured with CDC [50]. The figure demonstrates a clear separation between the light-velocity particles and the particle below threshold. The figure also shows the good agreement between data and Monte Carlo expectations [71].

3.2.5 Time of Flight Counter and Trigger Scintillation Counter

The *Time of Flight Counter* (TOF) provides is another device for the particle identification. For a flight path of 1.2 m, TOF with ~ 100 ps time resolution provides the π/K separation effective for the particle momentum range. To identify the particle, each particle mass (m) is measured as follows:

$$m = p \cdot \sqrt{(T/L)^2 - 1},$$

where L is the flight path length and T is the flight time. A coincidence between the *Trigger Scintillation* Counters (TSCs) and TOF Counters provides the clean event timing to the Belle trigger system. The TOF/TSC



Figure 3.10: Arrangement of the ACC.



Figure 3.11: Schematic drawing of a typical ACC counter module.



(a) Pulse-height spectra for 3.5 GeV pions and protons obtained by a single counter module of ACC, of which silica aerogel radiator has refractive index of 1.015, in a magnetic field of 1.5 T.

(b) Pulse-height spectra observed the barrel part for electrons and kaons identified by CDC dE/dx and TOF information. The Monte Carlo expectations are superimposed.

Figure 3.12: Performance of ACC

system also provides the fast trigger signal to SVD. The TOF/TSC system comprises of 64 TOF/TSC modules. The structure of TOF/TSC modules consists of two TOF counters and a TSC as shown in Figure 3.13. The TOF counters and TSCs are made of plastic scintillation counters and FM-PMTs for readout. The TOF counters has sensitive region of ~ 2.5 m long and covers the polar angle range of $34^{\circ} < \theta < 121^{\circ}$ where the TOF counters and TSCs are located at r = 122.0 cm and r = 117.5 cm, respectively. TOF is calibrated with $e^+e^- \rightarrow \mu^+\mu^-$ events [52]. Figure 3.14(a) shows the measured time resolution for forward, backward PMTs and the weighted average of both ends, as a function of the track position in the beam direction. The resolution for weighted average time is required to be ≤ 100 ps. Figure 3.14(b) shows the mass distributions from TOF measurements, for particles momenta below 1.2 GeV/c, in hadronic events with the Monte Carlo expectation by assuming the time resolution of 100 ps [72]. Peaks corresponding to pions, kaons and protons are significantly separated and the experimental data and Monte Carlo expectation are in good agreement.

3.2.6 Electromagnetic Calorimeter

The Electromagnetic Calorimeter (ECL) is designed to detect the electromagnetic shower produced by photons and electrons. The electromagnetic shower, which can be associated with the charged track reconstructed in CDC, is recognized as the electrons. The remaining showers are regarded as the neutral particles, i.e. photons. The electron identification by the ECL is essential for the efficient flavor identification of B mesons and for the reconstruction of J/Ψ in CP eigenstates such as $B^0 \rightarrow J/\Psi K_S$. Therefore, the good energy resolution is required for the hadron rejection in the election identification. The detail of the electron identification is described in Section 3.4.2. It is important to detect high-energy photons up to ~ 4 GeV to reconstruct the interesting rare decays, such as $B \rightarrow K^*\gamma$, $B^0 \rightarrow \pi^0\pi^0$ and so on. On the other hand, it is also required to have the good sensitivity for low energy photons ($\leq 500 MeV$), since the most of photons comes from π^0 in the cascade B-meson decays.

The overall configuration of ECL is shown in Figure 3.15. ECL consists of 8736 thallium doped cesium iodide (CsI(Tl)) crystals assembled into a tower structure. The barrel crystals are located at r = 1.15 m, while the forward and backward endcap crystals are located at z = +1.96 m and z = -1.02 m, respectively. The polar angular coverage is $17^{\circ} < \theta < 150^{\circ}$ corresponding to 91% of full solid angle. The numbers of crystals in barrel, forward endcap and backward endcap are 6626, 1152, and 960, respectively. Each CsI(Tl) crystal is 30 cm long corresponding to $16.2X_0$, where X_0 represents the radiation length. Each crystal is projected to the interaction point with a small tilt angle of 1.9° in both θ and ϕ directions to prevent any photons to escape trough the gaps



(b) Dimensions of TOF/TSC module

Figure 3.13: Structure of the TOF/TSC module. The lengths in the figure are in units of mm.



(a) Time resolution of TOF for $e^+e^-\to\mu^+\mu^-$ events as function of track position in beam direction.

(b) Mass distribution from TOF measurements for particles momenta below 1.2 GeV/c. The points shows the date obtained from hadronic events, while the histogram shows the expectation from Monte Carlo by assuming time resolution is 100 ps.

between crystals. Each crystal is readout by $2.10 \times 20 \text{ mm}^2$ photodiodes, which are glued to the rear surface of the crystal via 1 mm thick acrylate for the protection of the crystal.

The photon reconstruction is initiated by finding the ECL cluster. At first, the *seed crystals*, which have the highest energy deposit than any neighbor crystals are searched. The energy sum in 3×3 crystal matrices (E_9) and in 5×5 crystal matrices (E_{25}) around the seed crystals are calculated, where the crystals with the energy deposit of less than 500 KeV are excluded from the sum. To reject the cluster due to the charged particle, the crystal surface of the cluster must not be associated with any charged particle trajectory. The clusters with E_{25} greater than 500 MeV are identified as photons. The clusters with E_{25} greater than of 20 MeV and $E_9/E_{25} > 0.75$ are also identified as photons.

Prototype CsI crystals are tested at the tagged photon beam at Budker Institute of Nuclear Physics (BINP), Russia [53, 73, 74]. Figure 3.16(a) shows the average position resolution as a function of the measured cluster energy. The measured position resolution is expressed as $(0.27 + 3.4 \cdot E^{-1/2} + 1.8 \cdot E^{-1/4})$ mm, where E is the measured cluster energy in GeV. Figure 3.16(b) shows the energy resolution as a function of the incident photon energy. The energy resolution is measured as $(1.34 \oplus 0.066 \cdot E^{-1} \oplus 0.81 \cdot E^{-1/4})$ % where E is the incident photon energy. The absolute energy calibration is carried out using Bhabha and $e^+e^- \rightarrow \gamma\gamma$ events. Figure 3.17 shows the measured energy resolution for Bhabha events. The obtained energy resolutions are 1.70 % for barrel, and 1.74 % for the forward endcap and 2.85 % for the forward and backward endcap, respectively.



Figure 3.15: Overall structure of the ECL

3.2.7 Super-conducting Solenoid

A superconducting solenoid provides a magnetic field of 1.5 T in a cylindrical volume of 3.4 m in diameter and 4.4 m in length for the charged-particle tracking. The main coil is made of the niobium-titanium/copper superconductor with pure aluminum stabilizer. The coil is chilled by liquid helium and has inductance of 3.6 H. The nominal current is 4,400 A. The return path of the magnetic field is provided with the iron yoke that also works as the absorber material for the detection of muons and K_L mesons. An absolute calibration of the field strength with a precision better than 1 Gauss is provided by the NMR probe.

3.2.8 K_L and Muon Detector

The K_L and Muon Detector (KLM) [56] is located outside the solenoid and designed to detect muons in the momentum range of > 0.6 GeV/c and K_L mesons. The muon identification is important to reconstruct of J/ψ mesons and the flavor tagging, as same as the electron identification. The detection of K_L mesons is important to reconstruct $B^0 \rightarrow J/\psi K_L$ decays. Although KLM does not measure the energy of K_L , $B^0 \rightarrow J/\psi K_L$ decays are reconstructed with adequate S/N ratio by requiring the kinematical constraints assuming two-body decays. KLM consists of barrel, forward endcap and backward endcap regions. The structure of each part is a repetition of 47 mm thick iron plate and 44 mm thick slot in which a *Resistive Plate Counter* (RPC) [75] super-layer module


(a) The average position resolution as a function of the measured cluster energy. The solid line shows the fit result.

(b) The Energy resolution as a function of incident photon energy.





Figure 3.17: Energy resolution of ECL measured with Bhabha events: overall, barrel, forward endcap and backward endcap.

is installed. There are 15 super-layers in the barrel and 14 layers in the each endcap. The barrel is divided azimuthally into octants and the endcap divided into quadrants.

The RPC super-layer module consists of two RPC layers as shown in Figure 3.18(a). A RPC consists of two highly resistive glass electrodes ($\geq 10^{10} \ \Omega \cdot cm$) with the gap of 2mm in which the gas made of 30 % argon, 8 % butane-silver⁵ and 62 % freon⁶ is filled. High voltage of +4.7 KV, +4.5 KV and -3.5 kV is supplied to the cathodes of barrel RPCs, the cathodes of endcap RPCs and the anode of all RPCs, respectively. The avalanche is induced by an incident charged particle in the gas. It results in a local discharge of the plate. Because of high resistivity of the plate and the quenching effect of the iso-butane and freon, the discharge is localized and readout as a signal with a good position resolution by pickup strips made of copper. Since strips of two RPC layers in a super-layer module are perpendicular to each other, the super-layer module can measure the position of charged particles in 2-dimensionally.

The barrel part consists of 240 super-layer modules that are rectangle in shape with external thickness, width, and length of 33 mm, $1542 \sim 2697 \text{ mm}^7$ and 2207 mm, respectively. Two barrel super-layer modules are aligned along the beam direction in one slot. Each endcap part consists of 112 super-layer modules as shown in Figure 3.18(b). The KLM covers the polar-angle region of $20^\circ < \theta < 155^\circ$. The signals from RPCs are fast (~20 ns) and are transferred to the trigger system.

The muon detection efficiency is measured by cosmic-ray muons, where the momenta are measured by CDC with a magnetic field of 1.5 T, as shown in Figure 3.19(a). The low momentum muons below 0.5 GeV/c do not reach the KLM. The method of muon identification is described in Section 3.4.3.

In order to identify K_L mesons, all charged tracks measured in CDC are extrapolated to KLM. The clusters within 15 degrees from extrapolated charged tracks are excluded, and the remaining KLM clusters are identified as K_L s. The direction of the K_L meson is determined by the center-of-gravity of the hits in the cluster. Figure 3.19(a) shows the difference between the neutral cluster and the direction of the missing momentum in the beam-collision data. Although a large deviation due to undetected neutrinos and the particle missing due to the detector acceptance, the peak corresponding to K_L candidates is significantly observed.



(a) The cross section

(b) The cut-away view of an endcap module

Figure 3.18: The KLM RPC super-layer module.

⁵A mixture of ~ 70 % n-butane (C₄H10) and ~ 30 % iso-butane (C₄H10).

 $^{^{6}}$ CH₂FCF3 (HFC-134a)

⁷The width is varies depending on the radial position of the module.



(a) The muon detection efficiency as a function of momentum in cosmic ray data.

(b) The difference between the neutral cluster ant the direction of missing momentum in beam collision data.

Figure 3.19: Performance of KLM

3.2.9 Extreme Forward Calorimeter

Extreme Forward Calorimeter (EFC) [57,58] extends the polar angle coverage for the electromagnetic shower in order to improve the experimental sensitivity to some physics processes such as $B \to \tau \nu$. EFC can also be used as a tagging device for two-photon physics. EFC covers the angular range from 6.4° to 11.5° in forward direction and 163.3° to 171.2° in the backward direction. Since EFC is placed in the very high radiation-level area around the beam pipe near the interaction point, EFC consists of a crystal calorimeters made of a BGO (Bi₄Ge₃O₁2) [76–78] that satisfies the requirement for the radiation hardness. Both forward and backward EFC consists of the BGO crystals segmented into 5 regions in the θ direction and 32 regions in ϕ direction. Typical cross-section of a crystal is about 2 × 2 cm² with 12 (10.5) radiation lengths for forward (backward), which have the energy resolution of 7.3% (5.8%) in RMS for forward (backward). EFC also work as a beam mask to reduce the backgrounds for CDC. In addition, EFC provides a fast online feedback about the luminosity and the beam condition like background rates to KEKB operation system.

3.2.10 Trigger System

The trigger system is required to reduce the beam-background rate within the tolerance of the data acquisition system, 500 Hz maximum, while the efficiency for physics events of interest is kept high. Expected trigger rates for various processes with a luminosity of 10^{34} cm⁻²s⁻¹ are listed in Table 3.2. Total trigger rate for physics processes is estimated as about 100 Hz. On the other hand, high beam backgrounds are expected because of the high beam current. Based on simulation studies, we expect about 100 Hz beam-related backgrounds that are dominated by spent electrons and positrons, which go out the beam orbit due to the bremsstrahlung and the beam background from the interactions between the electron/positron beam and the residual gas molecules in the beam chamber. The trigger system required to be robust against unexpectedly high beam background rates because beam backgrounds are very sensitive to accelerator conditions, and difficult to estimate reliably.

The trigger system in Belle experiment consists of the Level-1 (L1) hardware trigger [79, 80] and the Level-3 (L3) software trigger [81]. Figure 3.20 shows the schematic view of the L1 hardware trigger system. It consists of the sub-detector trigger systems and the central trigger system called the *Global Decision Logic* (GDL). The sub-detector trigger systems are based on two categories: track triggers and energy triggers. CDC and TSC are used to yield trigger signals for charged particles⁸. CDC provides $r-\phi$ and r-z track trigger signals. The ECL trigger system provides triggers based on the total energy deposit and the cluster counting of crystal hits. These two categories allow sufficient redundancy. The KLM trigger gives additional information on muons and the EFC

 $^{^{8}}$ Note that the TSC trigger provide the fast Level-0 (L0) trigger signal to the SVD readout system with a latency of 0.85μ s.

triggers are used for tagging two-photon process events as well as Bhabha events. The sub-detector triggers process event signals in parallel and provide trigger information to GDL, where all information is combined to characterize an event type and make decision to initiate the data acquisition within 2.2 μ s from beam crossing. Typical L1 trigger rate is about 250 Hz with a luminosity of about 6.5×10^{33} cm⁻²s⁻¹ [82]. Typical L1 trigger rate normalized by the beam current is 110 Hz/A. The L3 software trigger is implemented in the online computer farm. In the L3 trigger, the beam background is rejected using the information of roughly reconstructed charged tracks. The reduction rate of L3 trigger is 38%, while the efficiency for the hadronic events is 99% [81].

Table 3.2: Expected trigger rates for various processes with a luminosity of $L = 10^{34} \text{ cm}^{-2} \text{s}^{-1}$. θ_{lab} is the azimuthal angle of the final state particle in the laboratory frame. p_t is the transverse momentum of the final state particle.

Process	Rate (Hz)	note
$e^+e^- \to \Upsilon(4S)$	12	
continuum: $e^+e^- \rightarrow q\bar{q}(q=u,d,s,c)$	28	
$e^+e^- \rightarrow \ell^+\ell^- (\ell=\mu,\tau)$	16	
Bhabha: $e^+e^- \rightarrow e^+e^-(\theta_{\text{lab}} > 17)$	4.4	(pre-scaled by factor $1/100$)
$e^+e^- \to \gamma\gamma(\theta_{\rm lab} > 17)$	0.24	(pre-scaled by factor $1/100$)
2 photon processes: $\gamma\gamma \rightarrow \text{anything}(\theta_{\text{lab}} > 17, p_t > 0.1 \text{ GeV})$	~ 15	
total	~ 96	



Figure 3.20: Schematic view of the Level-1 trigger system of Belle.

3.2.11 Data Acquisition System

The required performance for the *Data Acquisition System* (DAQ) is to work at the 500 Hz trigger rate with a deadtime fraction of less than 10%, under the high luminosity operation of KEKB. In order to achieve this requirement, the distributed-parallel system is devised. The global scheme of the system is shown in Figure 3.21 [83,84].

The entire system is segmented into seven subsystems running in parallel, and each handles the data from the associated sub-detector. The data from subsystems are combined into a single event record by the *event builder* [85], which converts *detector-by-detector* parallel data streams to an *event-by-event* data river. The event builder output is transferred to an online computer farm, where the event filtering by Level-3 trigger is done after the fast event reconstruction. The data are then sent to a mass storage system located at the computer center,

3.3. OFF-LINE COMPUTING

2 km apart from the Belle detector, via optical fibers. The online computer farm also sends the sampled events to data quality monitor, by which experimental shifters monitor the detector and DAQ condition. The all the Belle DAQ components are controlled based on the Ethernet connections.

We adopt a charge-to-time (Q-to-T) technique to read out the analog signals from the sub-detectors except for SVD and KLM. The Q-to-T modules convert the charge amount to the time interval. Then the time interval is digitized by the time digitization module (TDC) which is controlled by VME and FASTBUS system [86]. KLM provides the time-multiplexed signals on a single line. This KLM signals are recorded by TDC as the time pulses. Because SVD has large number of channels, the SVD signals are digitized by the intelligent flash ADC modules, which perform the data sparsification. Four CPU modules on a VME bus control the flash ADC modules and send the digitized data to the event builder [63].

A typical data size of a hadronic event is measured to be about 30 kB, which corresponds to the maximum data transfer rate of 15 MB/s. The typical dead time is about 4% at the trigger rate of 250 Hz. The overall error rate of the data acquisition flow is measured to be less than 0.05%.



Figure 3.21: Schematic view of the Belle DAQ system.

3.3 Off-line Computing

The collected data by the Belle detector is analyzed using the offline computer farm. The offline computer farm is also used do the Monte Carlo simulation study. The required processing power of the offline computers amounts to 15000 SPECint95s. Because this computational power cannot be achieved by a single CPU, we developed the parallel processing scheme by multi-CPUs. We choose the Symmetric Multi Processor (SMP) architecture as a platform. The total storage capability of the offline computer farm is ~ 10 TB.

We developed our own data processing frameworks called BASF/FPDA (*Belle Analysis Framework/Framework for Parallel Data Analysis*) [87,88], which are especially suitable for both the production of the data summary tape (DST) and the physics analysis. The recorded events are sequentially scanned by one process. Each bulk of around 10 events is distributed to another process. The number of processes is equal to the number of embedded CPUs on the SMP machine. The distributed bulks of the events are processed on CPUs in parallel, and then returned to another process to be stored onto a tape, a hard disk, or the other storage media. The branching output path for the HBOOK format [89] is also equipped. With the framework running on the offline computer farm, the production speed of the DST is ~ 1 fb⁻¹ per day.

The processing framework also provides a scheme of the Monte Carlo events generation. The bulks of seeds of a random numbers instead of events themselves are distributed to the generator processes running on CPUs. A decay simulator is a program that generates particles tracing the decay chains according to the given manuscripts. The initial state of the particle generation is typically chosen to be $\Upsilon(4S)$ or $q\bar{q}$. The event generator used in the Belle analysis is QQ98 originally developed by the CLEO group [90], and is modified by the Belle group [91]. GEANT [92] is used as a full detector simulation in the Belle. It was originally developed at CERN. The GEANT simulator takes the generated events by QQ98 and produces the hit records at each sub-detector associated to the generated particles. The outputs from the full simulator are stored in the same format as the real data so that one can use the same analysis programs for both real and simulated data samples.

3.4 Particle Identification

This section describes the particle identification methods. In general, we use a likelihood method to calculate the probability for the particle, by combining the information from one or more sub-detectors. At first, the likelihood for each discriminant is defined using the *probability density functions* (PDFs), $P^n(i)$, where the subscript *n* refers to the type of discriminant such as CDC dE/dx, TOF and so on, and *i* indicates the particle species, *e*, μ , π , *K* or *p*. Then the likelihood ratio combining these discriminants to distinguish the particle species *i*, from the others is defined as:

$$L(i) \equiv \frac{\prod_{n} P_n(i)}{\prod_{n} P_n(i) + \prod_{n} \{\sum_{j \neq i} P_n(j)\}}.$$
(3.1)

While the likelihood ratio to distinguish the particle species i, from another species j is defined as:

$$L(i;j) \equiv \frac{\prod_{n} P_n(i)}{\prod_{n} P_n(i) + \prod_{n} P_n(j)}.$$
(3.2)

We describe the detail in the following subsections.

3.4.1 Kaon identification

In the reconstruction of $B^0 \to \pi^+\pi^-$, the separation of kaons from pions is very crucial to reduce the contamination of $B \to K\pi$ decays. It is also important for the flavor tagging. To distinguish kaons from pions, we use the dE/dxmeasured with CDC and information from TOF and ACC. Here we calculate the PDF likelihood in each component, then obtain the likelihood ratio from Equation 3.1 in the flavor tagging [93].

The likelihood for the dE/dx measured with CDC is:

$$P_{dE/dx}(i) = \frac{\exp(-\chi^2/2)}{\sqrt{2\pi}\sigma_{dE/dx}},$$

$$\chi^2 = \left[\frac{(dE/dx)_{\text{mesured}} - (dE/dx)_i}{\sigma_{dE/dx}}\right]^2.$$

where *i* is the particle species (π, K) , $(dE/dx)_{\text{mesured}}$ and $(dE/dx)_i$ are the measured dE/dx and the expected value for the species *i*, respectively, and $\sigma_{dE/dx}$ is an expected resolution of dE/dx.

The likelihood for TOF is:

$$P_{\text{TOF}}(i) = \frac{\exp(-\chi^2/2)}{\prod_k \sqrt{2\pi\sigma_{k,i}}},$$

$$\chi^2 = \sum_k (\boldsymbol{t}_{\text{measured}}^k - \boldsymbol{t}_i^k)^T E^{k^{-1}} (\boldsymbol{t}_{\text{measured}}^k - \boldsymbol{t}_i^k)$$

where the subscript k expresses the k-th TOF hits, $\mathbf{t}_{\text{mesured}}$ and \mathbf{t}_i are 2-dimensional vector containing the measured times in the two PMTs at the each end of TOF counter, and expected value, respectively, E is the 2 × 2 error matrix for ($\mathbf{t}_{\text{measured}} - \mathbf{t}_i$) and σ_k is the expected time resolution.

As described in Section 3.2.4, ACC is basically an on-off device, where the observed number of photons-electrons, $N_{\text{p.e.}}$ is either zero for heavy particles such as kaons, or the finite number for pions according to small-number statistics. In reality, even the distribution of $N_{\text{p.e.}}$ for kaons or protons peaks at zero, but the tail part is not negligible as shown in Figure 3.12. This tail is considered to be caused by scintillation light from the reflector, high energy δ -rays⁹ and the noise on readout electronics. To take into account this effect, the likelihood for ACC, $P_{\text{ACC}}(i)$, is parameterized using $N_{\text{p.e.}}$ [51]. $P_{\text{ACC}}(i)$ for the particle species *i* is obtained from the distribution of

 $^{^{9}}$ An orbital electrons knocked on from the counter material by an incident particle.

3.4. PARTICLE IDENTIFICATION

 $N_{\text{p.e.}}$ in the Monte Carlo simulation. $P_{\text{ACC}}(i)$ is defined with the dependence of the velocity of charge tracks for each counter module.

The performance of the kaon identification is evaluated using D^* -tagged $D^0 \to K^-\pi^+$ decays. Figure 3.22(a) shows the scatter plot of the obtained likelihood ratio vs. the track momenta for kaons and pions. The kaons are successfully separated from the pions and vise versa. Figure 3.22(b) shows the kaon identification efficiency and wrong identification fraction to pion as kaon for the selection of $L(k;\pi) \ge 0.6$ as a function of the track momentum. For the selection of $L(k;\pi) \ge 0.6$, the average efficiency is 88.7 % and the average fraction of the pion contamination is 8.53 %.

The likelihood ratio for the reconstruction of two-body decays of B mesons such as $B^0 \to \pi^+ \pi^-$, $L^{hh}(K;\pi)$, is constructed by the dE/dx measured with CDC and ACC information only because the particles from the two-body B meson decays have the high momenta above the sensitive momentum range of TOF. Thus,

$$P_{\text{combined}}^{hh}(i) = P_{\text{d}E/\text{d}x}(i) \cdot P_{\text{ACC}}(i),$$

$$L^{hh}(K;\pi) = \frac{P_{\text{combined}}^{hh}(K)}{P_{\text{combined}}^{hh}(K) + P_{\text{combined}}^{hh}(\pi)}$$

Figure 3.23 shows the performance of the kaon identification by CDC and ACC measured using D^* -tagged $D^0 \to K^-\pi^+$ decays, where the track momentum range is limited from 2.4 GeV/c to 2.85 GeV/c in the cms, corresponding to the kinematic region of the two-body *B* meson decays. Figure 3.23(a) shows $L^{hh}(K;\pi)$ is useful discriminant for the reconstruction of the two-body *B* meson decays. Figure 3.23(b) and Figure 3.23(c) show the efficiencies for pions and kaons as a function of the polar angle of the track in cases: $L^{hh}(K;\pi) \leq 0.4$ and $L^{hh}(K;\pi) \geq 0.6$, respectively. Here the Monte Carlo expectations are also plotted. The experimental data agree with the Monte Carlo expectation. The average efficiencies for the criterion: $L^{hh}(K;\pi) \leq 0.4$ ($L^{hh}(K;\pi) \geq 0.6$) for pions and kaons are 86% (9.3%) and 5.8% (92%), respectively.



(a) The likelihood ratio, $L(K;\pi)$, vs. the momentum of tracks. Open circles and filled circles show the kaons and pions, respectively.

(b) The kaon identification efficiency and wrong identification fraction to pion as kaon for the selection of $L(k;\pi) \ge 0.6$.

Figure 3.22: Performance of the kaon identification by the CDC, the TOF and the ACC for the barrel region measured using D^* tagged $D^0 \to K^- \pi^+$ decays.

3.4.2 Electron Identification

For the electron identification, we use the information from ECL, CDC and ACC [55, 94]. The same likelihoods for the dE/dx and the ACC hits as that for the K/π separation described in Section 3.4.1 are used. The three



(a) The distribution of likelihood ratio, $L^{hh}(K;\pi)$, for kaons (solid line) and pions (broken line).

(b) The efficiency of the selection, $L^{hh}(K;\pi) \leq 0.4$, as a function of the polar angle of the track for pions (above) and kaons (below).

(c) The efficiency of the selection, $L^{hh}(K;\pi) \geq 0.6$, as a function of the polar angle of the track for pions (above) and kaons (below).

Figure 3.23: Performance of the kaon identification by CDC and ACC measured using D^* tagged $D^0 \to K^-\pi^+$ decays, where the track momentum range is limited from 2.4 GeV/c to 2.85 GeV/c in the cms.

discriminants using the ECL information are also used to derive the likelihood ratio for electrons.

The most powerful discriminant for the electron identification is the E/p ratio of the energy (E) measured by ECL and the momentum (p) measured by CDC. Since the mass of electrons is negligiblely small in the energy range of the interest, and electrons deposit almost all energy in ECL, $E/p \sim 1$ is expected for electrons. On the other hand, for the pions and other hadrons deposit MIP energy in ECL, then their E/p became smaller than one and the distribution of E/p broaden. Figure 3.24(a) shows the E/p distributions for the electrons in radiative Bhabha events and pions from $K_S \to \pi^+\pi^-$ decays. The difference between electrons and pions is significantly large.

The position matching between the CDC tracks extrapolated to ECL and the ECL clusters contributes to the electron identification. The position resolution for electron showers is smaller than that of hadronic showers. The ECL cluster position is determined by the center-of-gravity of the ECL hits in the cluster. The matching χ^2 is defined as

$$\chi^2 \equiv \left(\frac{\Delta\phi}{\sigma_{\Delta\phi}}\right)^2 + \left(\frac{\Delta\theta}{\sigma_{\Delta\theta}}\right)^2,$$

where $\Delta \phi$ and $\Delta \theta$ are the difference between the cluster position and the extrapolated position of the track in azimuth and polar angle directions, respectively, while $\sigma_{\Delta\phi}$ and $\sigma_{\Delta\theta}$ are the widths obtained by fitting the $\Delta\phi$ and $\Delta\theta$ distributions for electron to Gaussian. The distributions of matching χ^2 for electrons and pions are shown in Figure 3.24(b).

The difference of the shape between electromagnetic and magnetic showers is significant discriminant. The shower shape in the transverse direction can be evaluated with the quantity: E_9/E_{25} , which is defined as the ratio of E_9 to E_{25} described in Section 3.2.6. The distributions of E_9/E_{25} foe electrons and pions are shown in Figure 3.24(c). Electrons exhibit a peak around 0.95 with relatively small low-side tail, while pions have more events in the lower E_9/E_{25} region.

The combined likelihood ratio to distinguish electrons from others, L(e), is constructed with these three ECL discriminants and $P_{dE/dx}(i)$ and $P_{ACC}(i)$ according to Equation 3.1. Figure 3.25(a) shows the efficiency with $L(e) \ge 0.5$ in radiative Bhabha events for the barrel region as a function of the track momentum. The average efficiency grater than 90% is achieved. Figure 3.25(b) shows the fraction of pions identified as electron with the selection: $L(e) \ge 0.5$ measured using $K_S \to \pi^+\pi^-$ decays. The fake ratio is kept less than 1 % for the momentum region above 1.0 GeV/c.

3.4.3 Muon Identification

The muon identification is based on the fact that the range of muons in KLM is lager than those for the other hadrons, due to the lack of the hadronic interaction. Muons penetrate more KLM RPC layers than the pions



Figure 3.24: The distributions of electron identification discriminants by ECL for electrons and pions. The distributions for electrons (solid line) are obtained from the radiative Bhabha events. The distributions for pions (broken line) are obtained from the $K_S \to \pi^+\pi^-$ decays in hadronic events.



(a) The efficiency in radiative Bhabha events for the barrel region as a function of the track momentum.

(b) The probability of pion contamination in the barrel regions measured using $K_S\to\pi^+\pi^-$ decays.

Figure 3.25: The performance of electron identification with $L(e) \ge 0.5$. The data for positive tracks and negative tracks are denoted by closed circles and opened squares, respectively.

that are the dominant background source for the muon identification because of their similar mass. The muon identification is valid for the tracks with momenta grater than 0.6 GeV/c, which can reach KLM.

The likelihood to identify muons is constructed with the number of the KLM layer penetrated by the track N_{hits} and the distance of the KLM hit position from the track [95]. The charged tracks are extrapolated to KLM region and the number of the KLM layers crossing the extrapolated track N_{crossing} is calculated. The difference $\Delta N \equiv N_{\text{crossing}} - N_{\text{hits}}$ is the most effective discriminant in the muon identification as shown in Figure 3.26(a). The distribution of ΔN for muons has a peak at zero while that for pions is broad. Because muons are scattered by KLM materials less than pions, the deviation of the KLM hits by muons from the extrapolated track trajectory in muon hypothesis is smaller than that for pions. The reduced χ^2 of the KLM hit positions is also provides the clear discrimination as shown in Figure 3.26(b). The likelihood ratio is obtained by with these two discriminants according to the Equation 3.1.

Figure 3.27 shows the muon identification performance as a function of the track momentum. The efficiency for muons with the momenta of > 1GeV/c obtained from $2\gamma \rightarrow \mu^+\mu^-$ processes is grater than 90% (80%) for the likelihood Ratio greater than 0.1(0.9). The wrong-identification fraction to pions with the momenta of > 1GeV/c obtained from $K_S \rightarrow \pi^+\pi^-$ decays is less than 8% (2%) for the likelihood Ratio greater than 0.1(0.9) [96].



(a) The difference between the number of the KLM layers crossing the extrapolated track and the number of the KLM layer penetrated by the track.

(b) The reduced χ^2 of the KLM hit positions.

Figure 3.26: The distributions of muon identification discriminants obtained from Monte Carlo simulation. The solid lines and the broken lines represent the distributions for muons and pions, respectively.



Figure 3.27: Performance of the muon identification as a function of the track momentum. The filled circles represent the efficiency measured using $2\gamma \rightarrow \mu^+\mu^-$ processes. The filled triangles represent the wrong identification fractions to pions measured using $K_S \rightarrow \pi^+\pi^-$ decays.

Chapter 4

Event Selection and Reconstruction

In this chapter, the procedure of the event selection and reconstruction of the physical quantities related to the CP violation measurement is described. We select hadronic events from the data sample collected by the Belle detector. We reconstruct B mesons from two charged tracks. We reduce the background based on the information of the particle identification and kinematics. Because the B meson is produced via $e^+e^- \rightarrow \Upsilon(4S) \rightarrow B\overline{B}$, the remaining tracks are the decay products of the accompanying B meson. The flavor of the B meson is determined by the properties of these remaining tracks. The decay point of B meson is determined by the kinematical fit using charged tracks.

4.1 Event Sample

The data sample used in this analysis is collected with the Belle detector from January 2000 to July 2002. The integrated luminosity accumulated on the $\Upsilon(4S)$ resonance in this period is 78.16 fb⁻¹, corresponding to 85.0±0.5 million $B\overline{B}$ pairs¹. The data is also accumulated with the 50 MeV lower the resonance energy to investigate the continuum background properties (off-resonance data). The integrated luminosity of this off-resonance data used in this study is 8.83 fb⁻¹. The branching fractions of the decays related to this analysis are listed in Table 4.1. The expected numbers of the neutral B mesons decaying to $\pi^+\pi^-$ and $K^{\pm}\pi^{\mp}$ in this data sample are about 370 and 1500, respectively.

mode	Branching ratio
$\Upsilon(4S) \to B^0 \overline{B}{}^0$	$48 \sim 50\%$
$B^0/\overline{B}{}^0 \to \pi^+\pi^-$	$(4.4 \pm 0.9) \times 10^{-6}$
$B^0 \to K^+ \pi^-$	$(1.74 \pm 0.15) \times 10^{-5}$

Table 4.1: Branching fractions of the decay modes related to this analysis [42].

4.2 Hadronic Event Selection

The data sample contains several processes other than \overline{BB} production such as *continuum* $e^+e^- \rightarrow q\bar{q}(q = u, d, s, c)$ processes, μ pair production, τ pair production, QED processes referring to Bhabha and radiative Bhabha processes and two-photon processes. The cross sections for these processes are listed in Table 4.2.

The data sample also contains cosmic rays, spent electrons that go out the beam orbit due to the bremsstrahlung and the beam background from the interactions between the electron/positron beam and the residual gas molecules in the beam chamber.

To suppress these uninteresting events and select hadronic events dominated by the $\Upsilon(4S)$ production and the continuum, we apply event selections as follows:

1. The number of the good charged track in an event is required to be more than two. The good charged track is defined to satisfy $|\delta r| < 2.0$ cm, $|\delta z| < 4.0$ cm and $p_t > 0.1 \text{GeV}/c$ where δr , δz and p_t represents the impact parameter to the nominal interaction point in the x-y plane and along the z-axis, and the transverse momentum, respectively.

¹Both $B^0\overline{B}^0$ and B^+B^- are included.

4.2. HADRONIC EVENT SELECTION

This requirement is effective to suppress the QED processes and the μ pair production. It also reduces the cosmic lays and the beam background because the tracks from these events come from arbitrary points while the tracks from hadronic events come from the nominal interaction point.

- 2. At least one good ECL cluster should be within the fiducial volume of $-0.7 < \cos \theta < 0.9$. The good ECL cluster is defined as the cluster having the energy more than 100 MeV. This requirement reduces the QED processes because the most clusters from QED processes have very shallow angles.
- 3. The total visible energy, $E_{\rm vis}$ calculated from the good tracks assuming the pion mass and good photons in an events has to satisfy

 $E_{\rm vis} \ge 0.2\sqrt{s},$

where \sqrt{s} represents the cms energy. The good photon is defined as the good ECL cluster within the CDC acceptance² with no associated tracks from the CDC.

4. The energy sum of the good ECL clusters within the CDC acceptance, $E_{\rm sum}$, is required to satisfy

 $0.1 < E_{\rm sum} / \sqrt{s} < 0.8.$

This requirement reduces the QED processes. In the QED processes, $E_{\rm sum}/\sqrt{s} \sim 1$, because the final state particles are only electrons and photons. If one of the electrons in the QED events falls in the gap of the calorimeter, this QED events could satisfy the above requirement. To suppress this event, we apply an additional requirement. An average ECL cluster energy is also required to be

 $E'_{\rm sum}/(\# \text{ of good ECL cluster}) < 1.0 \text{ GeV},$

where E'_{sum} is the energy sum of all good ECL clusters including the ones outside the CDC acceptance.

5. The momenta sum of the good tracks and good photons is required to be balanced in the z direction to eliminate the beam background:

 $|P_z| < 0.5\sqrt{s}.$

6. To reduce the beam background, the position of the *primary vertex*, that is formed by all good tracks, is required to satisfy

 $|r_{\text{vertex}}| < 1.5 \text{ cm}$ and $|z_{\text{vertex}}| < 3.5 \text{ cm}$,

where r_{vertex} and z_{vertex} represent the positions of the primary vertex in the r- ϕ plane and the z-axis, respectively.

7. The event is split into two hemispheres by a plane perpendicular to the event thrust axis. The invariant mass of tracks in each hemisphere is calculated assuming a pion mass. This invariant mass is basically equivalent to the invariant mass of τ in τ pair production processes. In the event, we regard the larger invariant mass as heavy jet mass, M_{jet} . The events are required to satisfy

$$M_{\rm jet} > 1.8 \ {\rm GeV}/c^2 \quad {\rm or} \quad \begin{cases} E_{
m sum}'/\sqrt{s} > 0.18 \\ M_{
m jet}/E_{
m vis} > 0.25. \end{cases}$$

The efficiency of the hadronic event selection is estimated using Monte Carlo simulation. The selection retains 99.1% of $B\overline{B}$ events and 79.5% of the continuum processes while reducing the contamination of the non-hadronic components to be less than 5% [97]. The remaining non-hadronic components mainly consist of $e^+e^- \rightarrow \tau^+\tau^-$ events where both τ s decay to 3-prongs, and the large q^2 two-photon events.

 $^{^217^\}circ < \theta < 150^\circ$

Table 4.2: Cross sections for various processes in e^+e^- collisions at the cms energy equals to 10.58 GeV [44]. θ_{lab} is the azimuthal angle of the final state particle in the laboratory frame. p_t is the transverse momentum of the final state particle.

Process	cross section (nb)
$e^+e^- \to \Upsilon(4S)$	1.2
continuum: $e^+e^- \rightarrow q\bar{q}(q=u,d,s,c)$	2.8
$e^+e^- \rightarrow \ell^+\ell^- (\ell=\mu,\tau)$	1.6
Bhabha: $e^+e^- \rightarrow e^+e^-(\theta_{\text{lab}} > 17^\circ)$	44.
2 photon processes: $\gamma \gamma \rightarrow \text{anything}(\theta_{\text{lab}} > 17^{\circ}, p_t > 0.1 \text{ GeV})$	~ 15

4.3 $B^0 \rightarrow \pi^+\pi^-$ Reconstruction

4.3.1 Reconstruction of B^0

 B^0 mesons decaying to two charged pions are fully reconstructed from two tracks. The tracks are required to come from the *Interaction Point* (IP) [98]:

dr < 0.1 cm and |dz| < 4.0 cm,

where dr and dz represent the impact parameters of the tracks to the IP in the x-y plane and along the z-axis, respectively.

Tracks which are positively identified as electrons are excluded. The similar rejection on muons or protons are not applied, because the Monte Carlo studies show that there is no gain in significance. The kaon tracks are suppressed using the likelihood ratio, $L^{hh}(K;\pi)$, constructed by dE/dx measured by CDC and ACC, which is described in Section 3.4.1. The pion tracks are required to have $L^{hh}(K;\pi) < 0.4$.

To select B^0 meson candidates, two kinematical quantities, the *Beam-energy Constrained Mass* (M_{bc}) and the *Energy difference* (ΔE) , are defined as follows:

$$M_{\rm bc} = \sqrt{(E_{\rm beam}^*)^2 - (\boldsymbol{p}_B^*)^2},$$

$$\Delta E = E_B^* - E_{\rm beam}^*,$$

where E_{beam}^* is the beam energy in the $\Upsilon(4S)$ rest frame, E_B^* and p_B^* are the energy and momentum of reconstructed B^0 in the $\Upsilon(4S)$ rest frame. E_{beam}^* is calibrated run-by-run using $B^0 \to D^{(*)+}\pi^-$ decays and $B^+ \to D^{(*)0}\pi^+$ decays [99].

The signal events are localized at $(M_{\rm bc}, \Delta E) = (5.28, 0.0)$ as shown in Figure 4.1(a). The events that satisfy $M_{\rm bc} > 5.2 \text{ GeV}/c^2$ and $-0.3 < \Delta E < 0.5 \text{ GeV}$ are selected, which also includes the sideband region for the background study. The reconstruction efficiency of the $B^0 \rightarrow \pi^+\pi^-$ is estimated about 75 % by Monte Carlo.

The major sources of the background in $B^0 \to \pi^+\pi^-$ are the continuum events and $B^0 \to K^+\pi^-$ decays where kaons are misidentified as pions. The continuum background is suppressed using the event shape parameters as described in Section 4.3.2.

Figure 4.1(b) shows the $(M_{\rm bc}, \Delta E)$ distribution for the $B^0 \to K^+\pi^-$ decays contributing to the background. The ΔE peak position is shifted by -0.045 GeV because of a pion mass assignment to the kaon track. The treatment of the contamination from $B^0 \to K^+\pi^-$ decays is described in Section 4.3.3.

The contributions from the B^0 meson decays other than $B^0 \to \pi^+\pi^-, K^+\pi^-$ decays and B^+ meson decays, shown in Figure 4.1(c) and Figure 4.1(d), respectively, can be separated kinematically, because the ΔE of these components is less than -0.1 GeV.

A Monte Carlo simulation shows non-hadronic components do not form the $B^0 \to \pi^+\pi^-$ candidates. Figure 4.2 shows the event display of a $B^0 \to \pi^+\pi^-$ candidate in the real data.

4.3.2 Continuum Background Suppression

The continuum events are suppressed using the difference in event topologies of the continuum events and \overline{BB} events. In the cms, \overline{BB} is produced at rest and the two B decay axes are uncorrelated. Thus, the event shape of \overline{BB} events is spherical. On the other hand, two light quarks in the continuum events are produced back-to-back with hadronizing along a single axis. Therefore, the event shape of the continuum events is jet-like.



(c) The contamination from the other B^{0} decays.

(d) The contamination from the B^+ decays.

Figure 4.1: The ΔE vs. $M_{\rm bc}$ for $B^0 \to \pi^+\pi^-$ candidate from $B^0 \to \pi^+\pi^-$ decays, $B^0 \to K^+\pi^-$ decays, the B^0 meson decays other than these two decay modes and B^+ meson decays in Monte Carlo. All the candidates are required to satisfy the continuum suppression requirement described in Section 5.2.4. The box shows signal region described in Section 4.3.3.



(a) Whole view in x-y view

(b) Enlarged view of ECL part in y-z view





(d) Enlarged view of SVD part in y-z view

Figure 4.2: The event display of a $B^0 \to \pi^+\pi^-$ candidate in the real data.

4.3. $B^0 \rightarrow \pi^+\pi^-$ RECONSTRUCTION

The Fox-Wolfram moment [100] is used to quantify the event shape. The *n*-th Fox-Wolfram moment is defined as

$$H_n \equiv \sum_{i,j} |\boldsymbol{p}_i^*| |\boldsymbol{p}_j^*| P_n(\cos \theta_{ij}), \qquad (4.1)$$

where $P_n(x)$ is the *n*-th Legendre polynomial, \mathbf{p}_i^* and \mathbf{p}_j^* is the cms momenta of *i*-th and *j*-th particles, respectively, and θ_{ij}^* represents the angle between the decay axes of the *i*-th and *j*-th particles³, and the sum is over all particles in the final state. The normalized second Fox-Wolfram moment defined as $R_2 \equiv H_2/H_0$ is ~ 1 for jet-like events and ~ 0 for spherical events.

In this analysis, the *Improved Fox-Wolfram moment* [101] is used. Equation 4.1 is divided into three components.

$$H_n = H_n^{\rm ss} + H_n^{\rm so} + H_n^{\rm oo} \tag{4.2}$$

$$H_n^{\rm ss} \equiv \sum_{i,j} |\boldsymbol{p}_i^*| |\boldsymbol{p}_j^*| P_n(\cos \theta_{ij}) \tag{4.3}$$

$$H_n^{\rm so} \equiv \sum_{j,k} |\boldsymbol{p}_j^*| |\boldsymbol{p}_k^*| P_n(\cos \theta_{jk}) \tag{4.4}$$

$$H_n^{\text{oo}} \equiv \sum_{k,l} |\boldsymbol{p}_k^*| |\boldsymbol{p}_l^*| P_n(\cos \theta_{kl})$$
(4.5)

where i and j are taken over the daughter particles of the reconstructed B mesons and k and l are taken over the remaining particles. We compose a six variable Fisher discriminant [102] as

$$S \equiv \sum_{n=2,4} \alpha_n \left(\frac{H_n^{\rm so}}{H_0^{\rm so}} \right) + \sum_{n=1}^4 \beta_n \left(\frac{H_n^{\rm so}}{H_0^{\rm so}} \right) \tag{4.6}$$

where the α_n and β_n are Fisher coefficients. The α_n and β_n are determined as shown in Table 4.3 by the Monte Carlo in order to maximize the separation of the continuum events and $B\overline{B}$ events. The reason why $H_n^{\rm ss}$ is not used in the Fisher discriminant is that $H_n^{\rm ss}$ is strongly correlated with $M_{\rm bc}$ and ΔE . $H_1^{\rm so}$ and $H_3^{\rm so}$ are also not used because they have strong correlations with $M_{\rm bc}$.

The flight direction of reconstructed B meson with respect to the beam axis in the $\Upsilon(4S)$ rest frame, $\cos \theta_B$, is useful to separate the continuum events. Because the spin and parity of $\Upsilon(4S)$ are $J^P = 1^-$, the distribution of $\cos \theta_B$ in the $B\overline{B}$ events is proportional to $(1 - \cos^2 \theta_B)$ while that of the continuum events is uniform.

These two discriminants, the event shape and the flight direction, are combined into a single likelihood ratio. The probability density function (PDF) of the improved Fox-Wolfram moment for $B\overline{B}$ events, $P_{\text{shape}}(S; B\overline{B})$, and that for continuum events, $P_{\text{shape}}(S; q\overline{q})$, which are parameterized using the bifurcated Gaussian functions, are derived from the Monte Carlo simulation as shown in Figure 4.3(a). The PDF of $\cos \theta_B$ for $B\overline{B}$ events, $P_{\text{f.d.}}(\cos \theta_B; B\overline{B})$, which is parameterized as an second order polynomial function, and that for continuum events, $P_{\text{f.d.}}(\cos \theta_B; q\overline{q})$, which is constant, are also derived from the Monte Carlo simulation as shown in Figure 4.3(b). The combined likelihood ratio, \mathcal{LR} , is defined as follows:

$$L(i) \equiv P_{\text{shape}}(S; i) \cdot P_{\text{f.d.}}(\cos \theta_B; i) \qquad (i = B\overline{B}, q\overline{q}),$$
$$\mathcal{LR} \equiv \frac{L(B\overline{B})}{L(B\overline{B}) + L(q\overline{q})}.$$

 \mathcal{LR} gives the clear separation as shown in Figure 4.3(c), in which the histograms show the \mathcal{LR} distributions for the $B^0 \to \pi^+\pi^-$ signal events and the continuum background events. If we require that \mathcal{LR} be greater than 0.825, 95% of continuum background events are removed while retaining 53% of the signal events, and the expected S/Nratio is 0.4. This criterion maximizes the $S/\sqrt{S+N}$ ratio assuming a branching fraction of $B^0 \to \pi^+\pi^- = 5 \times 10^{-6}$ and the efficiency of continuum events obtained using the sideband region. The reconstruction efficiency including this criterion is 31% for the $B^0 \to \pi^+\pi^-$ signal. In this analysis, the selection criteria with \mathcal{LR} are determined with respect to each region of the flavor tagging quality. All the candidates are divided into six regions by the event-by-event dilution factor due to the flavor tagging, r, which is described in Section 4.4. The \mathcal{LR} distributions

³i.e. $\cos \theta_{i,j}^* = (\boldsymbol{p}_i^* \cdot \boldsymbol{p}_j^*) / (|\boldsymbol{p}_i^*||\boldsymbol{p}_j^*|)$

are different for each r regions as shown in Figure 4.4. The selection criteria of \mathcal{LR} requirements are determined to maximize the *Figure-Of-Merits* (FOM), which is defined as

$$FOM = S(\pi\pi)/\sqrt{S(\pi\pi) + N(K\pi) + N(q\bar{q})},$$

where $S(\pi\pi)$, $N(K\pi)$ and $N(q\bar{q})$ represent the numbers of the $B^0 \to \pi^+\pi^-$ signal events, $B^0 \to K^+\pi^-$ background events and continuum background events, respectively. Because S/N ratios differ between in the higher \mathcal{LR} region and in the lower \mathcal{LR} region, the candidates are separated into 12 \mathcal{LR} -r regions summarized in Table 4.4 and treated separately. In the following sections, six regions of $\mathcal{LR} > 0.825$ ($\mathcal{LR} \leq 0.825$) are referred to as higher \mathcal{LR} regions (lower \mathcal{LR} regions).

Table 4.3: The coefficients of the improved Fox-Wolfram moment obtained from Monte Carlo.

term	coefficient	term	coefficient
		β_1	-1.42886
α_2	-3.84061	β_2	0.0815516
		β_3	1.58844
α_4	-0.820487	β_4	1.50104

r	Ĺ	CR
$0.000 < r \le 0.250$	(1) $0.525 < \mathcal{LR} \le 0.825$	(7) $0.825 < LR \le 1.000$
$0.250 < r \le 0.500$	(2) $0.525 < \mathcal{LR} \le 0.825$	$(8) \ 0.825 < LR \le 1.000$
$0.500 < r \le 0.625$	(3) $0.425 < \mathcal{LR} \le 0.825$	$(9) \ 0.825 < LR \le 1.000$
$0.625 < r \le 0.750$	(4) $0.425 < \mathcal{LR} \le 0.825$	$(10) \ 0.825 < LR \le 1.000$
$0.750 < r \le 0.875$	(5) $0.425 < \mathcal{LR} \le 0.825$	$(11) \ 0.825 < LR \le 1.000$
$0.875 < r \le 1.000$	(6) $0.325 < \mathcal{LR} \le 0.825$	$(12) \ 0.825 < LR \le 1.000$

Table 4.4: 12 \mathcal{LR} -r regions.



(a) The improved Fox-Wolfram moment.

(b) The flight direction of reconstructed B meson.

(c) The likelihood Ratio.

Figure 4.3: The discriminants for continuum background suppression. The solid and dashed lines represent the PDF for signal derived from the Monte Carlo and the PDF for continuum background derived from the real data in the sideband region. The open and filled circles are the data obtained from the $B^+ \rightarrow D^0 (\rightarrow K^- \pi^+)\pi^+$ decays and off-resonance data, respectively. The PDFs obtained from Monte Carlo show the good agreement with the data.



Figure 4.4: The likelihood ratio distributions of for six flavor tagging quality regions. The solid and dashed line histograms show the \mathcal{LR} distributions of the signal and continuum background events, respectively. Also shown are the figure-of-merits curves. The vertical lines indicate the minimum LR required.

4.3.3Yield extraction

The signal candidates are required that their flavors are determined and their decay vertices of B mesons are measured.⁴ The efficiencies of the flavor determination and the vertex measurement are estimated to be 99.8%and 88%, respectively. The remaining candidates consist of the $B^0 \to \pi^+\pi^-$ signals, $B^0 \to K^+\pi^-$ events due to imperfect particle identification and the continuum background. The signal yield in the higher \mathcal{LR} regions is extracted from the $M_{\rm bc}$ and ΔE distributions using a binned maximum likelihood fit method. The signal yield in the lower \mathcal{LR} regions is estimated from the yield in the higher \mathcal{LR} regions scaled by a factor estimated using Monte Carlo.

The shapes of $M_{\rm bc}$ and ΔE distributions for signal events and for $B^0 \to K^+\pi^-$ background events are modeled by a single Gaussian⁵. Because the $M_{\rm bc}$ width is dominated by the beam energy spread, and it is independent of the decay chains of B meson. A Monte Carlo study shows the $M_{\rm bc}$ distributions of $B^0 \to \pi^+\pi^-$, $B^0 \to K^+\pi^-$ and $B^+ \to \overline{D}^0 \pi^+$ can be modeled as a common single Gaussian:

$$P_{M_{\rm bc}}^{B\overline{B}}(M_{\rm bc}) = G(M_{\rm bc}; \mu_{M_{\rm bc}}^{B\overline{B}}, \sigma_{M_{\rm bc}}^{B\overline{B}}), \qquad (4.7)$$

$$\mu_{M_{\rm bc}}^{B\overline{B}} = 5279.1 \pm 1.1 \text{ MeV}/c^2,$$

$$\sigma_{M_{\rm bc}}^{B\overline{B}} = 2.62 \pm 0.04 \text{ MeV}/c^2,$$

where the peak position, $\mu_{M_{\rm bc}}^{B\overline{B}}$, and the width, $\sigma_{M_{\rm bc}}^{B\overline{B}}$, are obtained from $B^+ \to \overline{D}^0 \pi^+$ decay data (Table 4.5). The ΔE distribution of signal, $P_{\Delta E}^{\pi\pi}(\Delta E)$, and that for $B^0 \to K^+\pi^-$ background, $P_{\Delta E}^{\pi\pi}(\Delta E)$, are parameterized using a single Gaussian as follows:

$$P_{\Delta E}^{\pi\pi}(\Delta E) = G(\Delta E; \mu_{\Delta E}^{\pi\pi}, \sigma_{\Delta E}^{\pi\pi}), \qquad (4.8)$$
$$\mu_{\Delta E}^{\pi\pi} = -0.3 \pm 0.3 \text{ MeV},$$

$$\begin{aligned}
\sigma_{\Delta E}^{\pi\pi} &= 18.8 \pm 0.7 \text{ MeV}, \\
P_{\Delta E}^{K\pi}(\Delta E) &= G(\Delta E; \mu_{\Delta E}^{K\pi}, \sigma_{\Delta E}^{K\pi}), \\
\mu_{\Delta E}^{K\pi} &= -44.3 \pm 0.8 \text{ MeV}, \\
\sigma_{\Delta E}^{K\pi} &= 22.6 \pm 0.5 \text{ MeV}.
\end{aligned}$$
(4.9)

 $\mu_{\Delta E}^{\pi\pi}$ is also determined using $B^+ \to \overline{D}^0 \pi^+$ decay data. Since we calculate the energy of final state charged particle using a pion mass assumption, $\mu_{\Delta E}^{K\pi}$ is shifted. This shift is determined by Monte Carlo. $\sigma_{\Delta E}^{\pi\pi}$ and $\sigma_{\Delta E}^{K\pi}$ are determined using inclusive $D^0 \to K^-\pi^+$ decays requiring the D daughter particles to have a momentum range similar to $B^0 \to \pi^+\pi^-$ candidate particles (from 1.5 GeV/c to 4.5 GeV/c). Comparison between the D mass width in Monte Carlo events and data are used to scale the $B^0 \to \pi^+\pi^-$ and $B^0 \to K^+\pi^- \Delta E$ widths in Monte Carlo. The signal region is defined as 5.271 GeV/ $c^2 < M_{\rm bc} < 5.278$ GeV/ c^2 and $|\Delta E| < 0.057$ GeV, corresponding

to $\pm 3\sigma$ from the central values, as shown in Figure 4.5. The number of candidates in the signal region is 760 of which the 275 (485) events are in higher (lower) \mathcal{LR} regions.

It is confirmed that the $M_{\rm bc}$ and ΔE distributions of continuum background for the signal region are identical to those for the sideband region using the off-resonance data. It is also confirmed that the $M_{\rm bc}$ and the ΔE shapes of continuum background are independent of the particle identification requirements using the real data. Thus, to increase statistics, the $M_{\rm bc}$ and the ΔE shapes of continuum background are determined using the $B^0 \to \pi^+ \pi^$ candidates in the mass sideband region, which are selected without particle identification requirements.

The $M_{\rm bc}$ distribution of continuum background components, $P_{M_{\rm bc}}^{q\bar{q}}(M_{\rm bc})$, is modeled by the ARGUS kinematical threshold function [103, 104] as follows:

$$P_{M_{\rm bc}}^{q\bar{q}}(M_{\rm bc};\alpha) = (x/E_{\rm beam}^*)\sqrt{1 - (x/E_{\rm beam}^*)^2} \exp\left[\alpha \cdot (1 - (x/E_{\rm beam}^*)^2)\right]$$
(4.10)

where E_{beam}^* is the beam energy in the $\Upsilon(4S)$ rest frame ⁶. The parameter α is determined as -18.5 ± 1.1 using the candidate in the ΔE sideband region, which is shown in Figure 4.5.

The ΔE distribution of continuum background component, $P_{\Delta E}^{q\bar{q}}(\Delta E)$ is parameterized as a first-order Chebyshev polynomial function:

$$P_{\Delta E}^{q\bar{q}}(\Delta E;c_1) = 1.0 + c_1 \cdot (2\Delta E - \Delta E_{\max} - \Delta E_{\min}) / (\Delta E_{\max} - \Delta E_{\min})$$

$$\tag{4.11}$$

⁴The detail of the flavor determination and the vertex measurements are described in Section 4.4 and 4.5.

⁵In this paper, Gaussian is referred to as $G(x;\mu,\sigma)$ where $G(x;\mu,\sigma) = 1/\sqrt{2\pi\sigma} \exp[-(x-\mu)^2/2\sigma^2]$.

 $^{^{6}}$ The beam energy in this function is fixed at 5290.0 MeV

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where $\Delta E_{\text{max}} = 0.5 \text{ GeV}(\Delta E_{\text{min}} = -0.3 \text{ GeV})$ is the upper (lower) boundary of the fit region and c_1 is a parameter to represents the slope of the distribution. The parameter c_1 is determined as -0.17 ± 0.01 using the candidates in the M_{bc} sideband region indicated in Figure 4.5.

Figure 4.6(a) shows the $M_{\rm bc}$ distributions in ΔE signal region. The yields of $B\overline{B}$ components, $N(B\overline{B})$ is obtained as 154.0 $^{+16.0}_{-15.2}$ by fitting to the function as follows:

$$P_{M_{\rm bc}}(M_{\rm bc}) = N(B\overline{B}) \cdot P_{M_{\rm bc}}^{B\overline{B}}(M_{\rm bc}) + N_{M_{\rm bc}}(q\overline{q}) \cdot P_{M_{\rm bc}}^{q\overline{q}}(M_{\rm bc}), \qquad (4.12)$$

where the fit result is superimposed in Figure 4.6(a).

Figure 4.6(b) shows the ΔE distributions in $M_{\rm bc}$ signal region. The numbers of $B^0 \to \pi^+\pi^ (N'(B^0 \to \pi^+\pi^-))$, $B^0 \to K^+\pi^ (N'(B^0 \to K^+\pi^-))$, and continuum events $(N'(q\bar{q}))$ in the $-0.3 < \Delta E < 0.5$ GeV region are obtained by fitting the following function to the data:

$$P_{\Delta E}(\Delta E) = N'(B^0 \to \pi^+ \pi^-) \cdot P_{\Delta E}^{\pi\pi}(\Delta E) + N'(B^0 \to K^+ \pi^-) \cdot P_{\Delta E}^{K\pi}(\Delta E) + N'(q\bar{q}) \cdot P_{\Delta E}^{q\bar{q}}(\Delta E).$$
(4.13)

The results are given in Table 4.7. The sum of $N'(B^0 \to \pi^+\pi^-)$ and $N'(B^0 \to K^+\pi^-)$ is consistent with the $N(B\overline{B})$ obtained from the $M_{\rm bc}$ distribution. The number of $B^0 \to \pi^+\pi^ (N(B^0 \to \pi^+\pi^-)), B^0 \to K^+\pi^-(N(B^0 \to K^+\pi^-)),$ and continuum events $(N(q\bar{q}))$ in the signal box, which are obtained from fit results, are listed in Table 4.7.



Figure 4.5: Definition of the signal region and ΔE and $M_{\rm bc}$ sideband regions.

Table 4.5: The parameters of $M_{\rm bc}$ distributions for $B^0 \to \pi^+\pi^-$, $B^0 \to K^+\pi^-$ and $B^+ \to \overline{D}^0\pi^+$. $\mu_{M_{\rm bc}}$ and $\sigma_{M_{\rm bc}}$ represent the peak position and width of the single Gaussian, respectively.

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Mode		$\mu_{M_{\rm bc}}$ (MeV)	$\sigma_{M_{\rm bc}}$ (MeV)
$B^0 \to \pi^+\pi^-$	(Monte Carlo)	5278.6 ± 0.1	2.77 ± 0.07
$B^0 \to K^+ \pi^-$	(Monte Carlo)	5278.7 ± 0.1	2.80 ± 0.05
$D^+ \rightarrow \overline{D}^0 \pi^+$	(Monte Carlo)	5278.6 ± 0.1	2.75 ± 0.03
$D^+ \rightarrow D^- \pi^+$	(Real data)	5279.1 ± 0.1	2.62 ± 0.04

4.4 Flavor Tagging

In order to measure the CP asymmetry, it is necessary to determine the flavor of the B meson (B_{tag}) accompanying to the B meson reconstructed from $\pi^+\pi^-$. The flavor of B_{tag} is identified with the information of remaining tracks which are not used in $B^0 \to \pi^+\pi^-$ reconstruction. The processes utilized in the flavor tagging are listed in Table 4.8. The efficiency of the flavor tagging, ε_{tag} , is not 100% because of the decay processes with very little flavor information such as $b \to c\bar{c}s$, the inefficiency of particle identification and so on. The wrong tag fraction, w_{tag} , which is the probability to assign the wrong flavor, is not zero because of the mis-identification of the particles

Table 4.6: The parameters of ΔE distributions for $B^0 \to \pi^+\pi^-$, $B^0 \to K^+\pi^-$ and $B^+ \to \overline{D}^0\pi^+$. $\mu_{\Delta E}$ and $\sigma_{\Delta E}$ represent the peak position and width of the single Gaussian, respectively.

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Mode		$\mu_{\Delta E} \ ({ m MeV})$	$\sigma_{\Delta E} \ ({\rm MeV})$
$B^0 \to \pi^+\pi^-$	(Monte Carlo)	2.4 ± 0.6	16.4 ± 0.6
$B^0 \to K^+ \pi^-$	(Monte Carlo)	-41.8 ± 0.5	19.7 ± 0.5
$B^+ \rightarrow \overline{D}^0 \pi^+$	(Monte Carlo)	2.7 ± 0.2	13.8 ± 0.2
$D^+ \rightarrow D^- \pi^+$	(Real data)	-0.3 ± 0.3	16.1 ± 0.3



(a) The $M_{\rm bc}$ distributions in ΔE signal region. The broken line and the dotted line show the $B\overline{B}$ and continuum components, respectively.

(b) The ΔE distributions in $M_{\rm bc}$ signal region. The broken line, the dash-dotted line, the dotted line and the solid-line histogram show the $B^0 \rightarrow \pi^+\pi^-$, $B^0 \rightarrow K^+\pi^-$ background, continuum background and the other B meson decays, respectively.

Figure 4.6: Fit to the $M_{\rm bc}$ and ΔE distributions in the higher \mathcal{LR} region. The open circles and the solid-line curves represent the data and the fitted results, respectively.

		$N'(B^0\to\pi^+\pi^-)$	$N'(B^0\to K^+\pi^-)$	$N'(q\bar{q})$
	$-0.3~{\rm GeV} < \Delta E < 0.5~{\rm GeV}$	$105.2 \ ^{+15.5}_{-14.8}$	$56.2 {}^{+14.6}_{-13.8}$	$854.6 \ _{-37.8}^{+38.8}$
		$N(B^0 \to \pi^+\pi^-)$	$N(B^0 \to K^+ \pi^-)$	$N(q\bar{q})$
signal region	$-0.057~{\rm GeV} < \Delta E < 0.057~{\rm GeV}$	$104.9 \ ^{+15.5}_{-14.8}$	$39.8 \stackrel{+10.3}{_{-9.8}}$	$120.5 \substack{+5.5 \\ -5.3}$

Table 4.7: The yield of $B^0 \to \pi^+\pi^-$ signal, $B^0 \to K^+\pi^-$ background and continuum background obtained with the ΔE distribution.

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and so on. A non-zero value of w_{tag} results in a dilution of the observed asymmetry. For example, if the true numbers of B^0 and \overline{B}^0 are $n(B^0)$ and $n(\overline{B}^0)$, the corresponding true asymmetry is

$$\mathcal{A}_{CP}^{\text{true}} \equiv \frac{n(B^0) - n(\overline{B}^0)}{n(B^0) + n(\overline{B}^0)}.$$

With realistic flavor tagging, the observed numbers of B^0 and $\overline{B}{}^0$ are

$$n'(B^{0}) = \varepsilon_{\text{tag}} \cdot [(1 - w_{\text{tag}}) \cdot n(B^{0}) + w_{\text{tag}} \cdot n(\overline{B}^{0})]$$

and
$$n'(\overline{B}^{0}) = \varepsilon_{\text{tag}} \cdot [w_{\text{tag}} \cdot n(B^{0}) + (1 - w_{\text{tag}}) \cdot n(\overline{B}^{0})].$$

$$n'(B^0) = \varepsilon_{\text{tag}} \cdot [w_{\text{tag}} \cdot n(B^0) + (1 - w_{\text{tag}}) \cdot n(B^0)]$$

Thus, the corresponding observed asymmetry is

$$\mathcal{A}_{CP}^{\text{obs}} \equiv \frac{n'(B^0) - n'(\overline{B}^0)}{n'(B^0) + n'(\overline{B}^0)} = (1 - 2w_{\text{tag}}) \cdot \mathcal{A}_{CP}^{\text{true}},$$

where the dilution factor is $(1 - 2w_{\text{tag}})$. Since the statistical error of the measured asymmetry is inversely proportional to $\sqrt{\varepsilon_{\text{tag}}}$, the number of events required to observe the asymmetry for a certain statistical significance is proportional to $\varepsilon_{\text{tag}}^{\text{eff}} \equiv \varepsilon_{\text{tag}} \cdot (1 - 2w_{\text{tag}})^2$, which is called the *effective tagging efficiency*. The performance of the flavor tagging is evaluated by $\varepsilon_{\text{tag}}^{\text{eff}}$.

The two parameters, q and r, are defined in event by event basis to represent the tagging information. The parameter q corresponds to the sign of the *b*-quark charge, where q = +1 for \bar{b} and hence B^0/B^+ , and q = -1 for b and \overline{B}^0/B^- . The parameter r is an event-by-event flavor-tagging dilution factor that ranges from r = 0 for no flavor discrimination to r = 1 for unambiguous flavor assignment. q and r are determined for each event using the look-up table method constructed with a large statistics Monte Carlo. Thus, q and r for the certain event are given by

$$q \cdot r \equiv \frac{N(B^0) - N(\overline{B}^0)}{N(B^0) + N(\overline{B}^0)},$$

where $N(B^0)$ and $N(\overline{B}^0)$ are the numbers of B^0 s and \overline{B}^0 s in the Monte Carlo sample, which have the same track properties as the event. When r is well constructed, the parameter r is related to u_{tag} as follows:

$$r = 1 - 2w_{\text{tag}}.$$
 (4.14)

The events are categorized into six bins in r, and w_{tag}^{l} is determined using the real data for each r bin, where the subscript l is the index of the r bin.

Process	Track information
Semi-Leptonic decay: $\begin{cases} b \to c\ell^- \bar{\nu}_\ell \\ \bar{b} \to \bar{c}\ell^+ \nu_\ell \end{cases}$	The charge of the primary lepton with high momentum.
Cascade charm decay: $\begin{cases} b \to cX \to s(\to K^-/\overline{\Lambda})X\\ \overline{b} \to \overline{c}X \to \overline{s}(\to K^+/\Lambda)X \end{cases}$	The charge of the kaons. The flavor of Λs .
Cascade semi-leptonic decay: $\begin{cases} b \to cX \to s\ell^+ \nu_\ell X \\ \bar{b} \to \bar{c}X \to \bar{s}\ell^- \bar{\nu}_\ell X \end{cases}$	The charge of the secondary lepton with low momentum.
$\pi \text{ production: } \begin{cases} \frac{B^0/B^- \to D^{(*)+}/D^{(*)0} + \pi^- X}{\overline{B}^0/B^+ \to D^{(*)-}/\overline{D}^{(*)0} + \pi^+ X} \end{cases}$	The charge of the pion with high momentum.
$D^* \text{ production: } \begin{cases} \frac{B^0/B^- \to D^{*+}X, D^{*+} \to D^0\pi^+}{\overline{B}^0/B^+ \to D^{*-}X, D^{*-} \to \overline{D}^0\pi^-} \end{cases}$	The charge of the pion with low momentum (so-called slow-pio

Table 4.8: The processes used in the flavor tagging.

4.4.1 Flavor Tagging Method

The flavor tagging is proceeded in two stages. In the first stage (*track-level flavor tagging*), the flavor tagging parameters, q_i and r_i , where the subscript *i* represents that they are obtained from the *i*-th track, are calculated

from each daughter track of B_{tag} independently. In the second stage (*event-level flavor tagging*), the results from track-level flavor tagging are combined to determine q and r of the event. Figure 4.7 shows the schematic view of the flavor tagging algorithm.

At the track-level flavor tagging stage, each track from B_{tag} is classified into one of four categories: leptons, kaons, Λ baryons, and slow pions. q_i and r_i are obtained with the look-up table method using several flavor-tagging discriminants defined for each category, such as the track momentum. The tracks do not satisfy $|\delta r| < 2.0$ cm and $|\delta z| < 10.0$ cm are not used, where δr and δz are the impact parameters to the nominal interaction point in the $r-\phi$ and z directions, respectively. The daughter tracks from K_S candidates are not used. The daughter tracks from the Λ baryons, which are reconstructed with $\Lambda \to p\pi$ decays, are put into Λ baryon category together. The tracks identified as the photon-converted electrons ⁷ are not treated as the electron candidates nor the slow pion candidates.

In the lepton category, the flavor-tagging parameters are extracted using the charge of the high momentum leptons from the semi-leptonic B decays and the medium momentum leptons from the cascaded semi-leptonic D decays. The lepton category consists of the two sub-categories: electron-like category and muon-like category. The tracks with $p_{\ell}^* > 0.4 \text{ GeV}/c$, where p_{ℓ}^* is the cms momentum, are assigned to the electron category if the likelihood ratio to distinguish electrons from kaon is greater than 0.8. The tracks of which p_{ℓ}^* is greater than 0.8 GeV/c are assigned to the muon category if the likelihood ratio to distinguish muons from kaons is lager than 0.95. The charge of the track Q_{ℓ}, p_{ℓ}^* , the likelihood ratio in particle identification, L_{PID} , and the polar angle of track, $\cos \theta_{\text{lab}}$ are used as the discriminants. The kinematical variables, the *recoil mass*, M_{recoil} and the missing cms momentum, p_{missing}^* are also taken into account because they are useful to distinguish the semi-leptonic decays, where M_{recoil} is defined as the invariant mass formed by all B_{tag} tracks except the lepton.

The look-up table of the multi-dimensional likelihood with these six variables is constructed from the Monte Carlo for each sub-category. The number of bins for Q_{ℓ} , p_{ℓ}^* , L_{PID} , $\cos \theta_{\text{lab}}$, p_{missing}^* and M_{recoil} are 2, 11, 4, 6, 6 and 10, respectively. Thus, total number of bins in the look-up table is 31680. The flavor tagging parameters, which are stored for each bin, are calculated as:

$$q_i \cdot r_i \equiv \frac{N(B^0; Q_\ell, p_\ell^*, L_{\text{PID}}, \cos\theta_{\text{lab}}, p_{\text{missing}}^*, M_{\text{recoil}}) - N(\overline{B}^0; Q_\ell, p_\ell^*, L_{\text{PID}}, \cos\theta_{\text{lab}}, p_{\text{missing}}^*, M_{\text{recoil}})}{N(B^0; Q_\ell, p_\ell^*, L_{\text{PID}}, \cos\theta_{\text{lab}}, p_{\text{missing}}^*, M_{\text{recoil}}) + N(\overline{B}^0; Q_\ell, p_\ell^*, L_{\text{PID}}, \cos\theta_{\text{lab}}, p_{\text{missing}}^*, M_{\text{recoil}})}$$
(4.15)

where $N(B^0; Q_\ell, p_\ell^*, L_{\text{PID}}, \cos \theta_{\text{lab}}, p_{\text{missing}}^*, M_{\text{recoil}})$ and $N(\overline{B}^0; Q_\ell, p_\ell^*, L_{\text{PID}}, \cos \theta_{\text{lab}}, p_{\text{missing}}^*, M_{\text{recoil}})$ are the numbers of B^0 and \overline{B}^0 events of the bin into which the track is classified. The look-up tables of other track categories are constructed in the similar ways. When the multiple tracks are classified into the lepton category, the parameters of the track that gives the largest r_i value are passed to the event-level flavor tagging stage.

The slow pion category is intended to distinguish the charge of the D^* from B_{tag} decays. The tracks with cms momenta, p_{π}^* is less than 0.25 GeV/c are assigned to the slow pion category if its likelihood ratio to distinguish kaons from pions is less than 0.9. To distinguish the slow pions from the pions with low momenta from the decays other than the D^* decay, the angle between the direction of the track and the thrust axis of the B_{tag} decay tracks, $\cos \theta_{\text{thrust}}$ are used as a discriminants because the direction of the slow pion from D^* decays follows the direction of the D^* . The electrons from photon conversion and π^0 Dalitz decays are also the contamination of this category. Because these low momentum tracks cannot reach ECL, the likelihood ratio to distinguish pions from electrons by the dE/dx measurement is used as the discriminant. The discriminants in the slow pion category are the charge, momentum in the laboratory frame, a polar angle, $\cos \theta_{\text{thrust}}$ and likelihood ratio for π/e separation. The flavor tagging parameters, q_i and r_i are calculated with these five discriminants using the look-up table method similar to the lepton category. When the multiple tracks are classified into the slow pion category, the parameters of the track that gives the largest r_i value are passed to the event-level flavor tagging stage.

In the Λ baryon category, the Λ candidates are selected with the loose requirements of secondary vertex reconstruction. The discriminants in this category are the flavor of the Λ , the invariant mass of the reconstructed Λ , the angle between the Λ momentum vector and the direction of the Λ decay vertex point from the nominal interaction point, the position difference of the two daughter tracks at the Λ decay vertex in z direction, and whether or not the events contains K_S candidates. The reason for using the existence of K_S candidates as the discriminant is that K_S carries out the strangeness as same as Λ s and Kaons. The asymmetry in the numbers of the Λ candidates and $\overline{\Lambda}$ candidates due to the secondary protons produced in the detector is taken into account.

The tracks, which are not classified into the other categories, are assigned to the kaon category if its likelihood ratio to distinguish proton from kaons is less than 0.7. The discriminants in this category are the charge, cms momentum, likelihood ratio for K/π separation and polar angle of the track.

In the event-level flavor tagging stage, the results from four categories in track-level flavor tagging are combined using the look-up table method also. While $q \cdot r$ from the lepton categories, and that from the slow pion category

⁷The photon-converted electrons are selected by the invariant mass selection of $m_{e^+e^-} < 0.1 \text{ GeV}/c^2$.

4.4. FLAVOR TAGGING

are determined, the results from the Λ baryon category and the kaon category are yet to be combined. The Λ baryon category and the kaon category are correlated each other because both are intend to observe the sum of the strangeness of B_{tag} decays. Therefore, the results from the Λ baryon category and the kaon category are combined as:

$$q_{K/\Lambda} \cdot r_{K/\Lambda} \equiv \frac{\prod_{i} (1 + q_i \cdot r_i) - \prod_{i} (1 - q_i \cdot r_i)}{\prod_{i} (1 + q_i \cdot r_i) + \prod_{i} (1 - q_i \cdot r_i)},$$

where the subscript i runs over the Λ candidates and the tracks in the kaon category.

q and r for the event are obtained by the look-up table with three discriminants, $q_{\ell} \cdot r_{\ell}$, $q_{\pi_s} \cdot r_{\pi_s}$ and $q_{K/\Lambda} \cdot r_{K/\Lambda}$, where the numbers of bins are 25, 19 and 35, respectively. The look-up tables are constructed using the independent Monte Carlo sample from that used for the construction of look-up table in the track-level flavor tagging in order to avoid any bias from a statistical correlation between the two stages. The efficiency that is defined as the fraction of r > 0 events is 99.6% in Monte Carlo.

The events are classified into six groups of the flavor tagging quality according to r values: $0 < r \le 0.25$, $0.25 < r \le 0.5$, $0.5 < r \le 0.625$, $0.625 < r \le 0.75$, $0.75 < r \le 0.875$ and $0.875 < r \le 1$.



Figure 4.7: The schematic view of the flavor tagging method.

4.4.2 Measurement of Incorrect flavor assignment Probability

The performance of the flavor tagging and the wrong tag fraction, w_{tag}^{l} are evaluated using the real data [105,106]. The one *B* meson is reconstructed using the flavor specific decays, $B^{0} \rightarrow D^{*-}\ell^{+}\nu$, $B^{0} \rightarrow D^{(*)-}\pi^{+}$, $B^{0} \rightarrow D^{*-}\rho^{+}$ and their charge conjugates, then the flavor tagging is performed to the other *B* meson. The overall efficiency of the flavor tagging in the real data is 99.8%, which is consistent with the Monte Carlo. w_{tag}^{l} is obtained through the time-dependent $B^{0}-\overline{B}^{0}$ mixing oscillation using the fact that the observed mixing amplitude is diluted by a factor $(1-2w_{tag})$. The obtained results for each *r* region are listed in Table 4.9. The total effective efficiency obtained by summing over the six *r* regions is calculated as $28.8 \pm 0.6\%$.

In the $B^0 \to \pi^+\pi^-$ reconstruction, the event shape requirement is introduced as described in Section 4.3.2 while the look-up tables in the flavor tagging are constructed from the generic $B^0\overline{B}^0$ Monte Carlo and w_{tag}^l is measured using the other B meson decays with the only loose event shape requirement. The event shape requirement might affect w_{tag}^l in each r regions because our flavor tagging method uses variables related to the event shape such as the missing momentum.

The effect of $B^0 \to \pi^+\pi^-$ selection to w_{tag}^l is investigated by comparing generic $B^0\overline{B}{}^0$ and $B^0 \to \pi^+\pi^-$ Monte Carlo samples. Table 4.10 shows the results. Comparing the generic $B^0\overline{B}{}^0$, the fractions of each r regions in $B^0 \to \pi^+\pi^-$ changed due to the $\pi^+\pi^-$ event selection. Then, as a result, the effective tagging efficiency increases in $B^0 \to \pi^+\pi^-$. On the other hand, the w_{tag}^l in each r region remains the same within the statistical errors of the Monte Carlo or errors of the measurements using the data. Therefore, the u_{tag}^l measured by $B^0 \to D^{*-}\ell^+\nu$, $B^0 \to D^{(*)-}\pi^+$ and $B^0 \to D^{*-}\rho^+$ are used in the CP violation measurement in $B^0 \to \pi^+\pi^-$ decays. The possible effect due to the $B^0 \to \pi^+\pi^-$ event selections is considered as an additional error of w_{tag}^l , listed in Table 4.9.

Table 4.9: The w_{tag}^l for each r region measured with the $B^0 \to D^{*-}\ell^+\nu$, $B^0 \to D^{(*)-}\pi^+$ and $B^0 \to D^{*-}\rho^+$ decays in real data. The total error of w_{tag}^l is the quadratic sum of statistical error, systematic error and the possible effect of the $B^0 \to \pi^+\pi^-$ selection estimated from Monte Carlo(Table 4.10).

l	r	w_{tag}^{l} (measured)	$w_{\rm tag}^l$ total error
1	0.000 - 0.250	$0.458 \pm 0.005 (stat) \pm 0.003 (syst)$	± 0.007
2	0.250 - 0.500	$0.336 \pm 0.008 (stat) \pm 0.004 (syst)$	± 0.010
3	0.500 - 0.625	$0.228 \pm 0.009 (\text{stat}) \begin{array}{c} +0.004 \\ -0.006 (\text{syst}) \end{array}$	$^{+0.011}_{-0.012}$
4	0.625 - 0.750	$0.160 \pm 0.007 (\text{stat}) \stackrel{+0.005}{-0.004} (\text{syst})$	± 0.014
5	0.750 - 0.875	$0.112 \pm 0.008(\text{stat}) \pm 0.004(\text{syst})$	± 0.015
6	0.875 - 1.000	$0.020 \begin{array}{c} +0.005 \\ -0.004 (\text{stat}) \end{array} \begin{array}{c} +0.005 \\ -0.004 (\text{syst}) \end{array}$	$^{+0.008}_{-0.007}$

Table 4.10: The comparison of event fractions and w_{tag}^l for each r region for generic $B^0\overline{B}^0$ Monte Carlo without any selection and $B^0 \to \pi^+\pi^-$ Monte Carlo events selected with the same selection in this analysis.

		generic B^{0}	${}^{0}\overline{B}{}^{0}$ Monte Carlo	$B^0 \to \pi^+ \pi^-$	Monte Carlo	
l	r region	fraction	$w_{ ext{tag}}^{l}$	fraction	$w_{ ext{tag}}^{l}$	difference of w_{tag}^l
1	0.000 - 0.250	0.420	0.472 ± 0.001	0.406 ± 0.002	0.471 ± 0.003	-0.001 ± 0.003
2	0.250 - 0.500	0.151	0.313 ± 0.002	0.149 ± 0.001	0.313 ± 0.005	$+0.000 \pm 0.005$
3	0.500 - 0.625	0.101	0.205 ± 0.002	0.104 ± 0.001	0.204 ± 0.005	-0.001 ± 0.005
4	0.625 - 0.750	0.114	0.141 ± 0.001	0.115 ± 0.001	0.131 ± 0.004	-0.010 ± 0.004
5	0.750 - 0.875	0.087	0.095 ± 0.001	0.093 ± 0.001	0.084 ± 0.004	-0.011 ± 0.004
6	0.875 - 1.000	0.125	0.027 ± 0.001	0.132 ± 0.001	0.025 ± 0.002	$+0.002 \pm 0.002$
	ϵ_{eff}	0.2	286 ± 0.001	0.305	± 0.003	

4.5 Reconstruction of Proper Time difference

In this section, we describe the reconstruction of the proper time difference:

$$\Delta t \equiv t_{CP} - t_{\text{tag}}, \tag{4.16}$$

where t_{CP} and t_{tag} are the decay time of B_{CP} and B_{tag} , respectively. Because B mesons are produced nearly at rest in the $\Upsilon(4S)$ rest frame and $\Upsilon(4S)$ is boosted along the z direction, the proper time difference is measured as

$$\Delta t = \frac{z_{CP} - z_{\text{tag}}}{(\beta \gamma)_{\Upsilon(4S)} \cdot c}, \tag{4.17}$$

where z_{CP} (z_{tag}) is the decay vertex position of B_{CP} (B_{tag}) in the z direction and ($\beta\gamma$)_{\Upsilon(4S)} is the Lorentz boost factor of Υ (4S). Thus, it is necessary to reconstruct the decay vertex positions of B_{CP} and B_{tag} and to calculate the vertex difference in the z direction, $\Delta z \equiv z_{CP} - z_{\text{tag}}$. In the time-dependent CP violation measurement, it is crucial to understand the resolution of Δt well. Therefore, the resolution of B decay vertexes should be well understood. To eliminate poorly reconstructed vertices, we reject a small fraction (~0.2%) of the events by requiring |dt| < 70 ps (~45 τ_{B^0}).

The decay vertices of B_{CP} and B_{tag} are reconstructed from the daughter charged tracks and the *interaction* point profile, which is described in Section 4.5.1, using the *kinematic fitting method* [107–109]. Figure 4.8 shows the conceptual drawing of the vertex reconstruction. The parameters of daughter tracks are re-fitted by requiring

to go through a certain point which is consistent with the expected *B* meson decay point using the least χ^2 method with Lagrange multiplier technique (*IP constraint*). The error of the decay point is calculated from the error of the track momentum and position, which are calibrated using cosmic ray tracks taking into account the difference between the tracks in hadronic events and cosmic ray tracks [110, 111].



Figure 4.8: The conceptual drawing of the measurement of the proper time difference.

4.5.1 Reconstruction of *B* Decay Position

In this section, the measurement of *interaction point profile* (IP profile) and the estimation of the *B* meson decay points are described. The IP profile is the over wrap region of the HER and LER beam bunches, as illustrated in Figure 4.8. The IP profile is slanted due to the finite crossing angle. The accelerator condition varies injection by injection of the beam. It also changes even during the same run. Therefore, if the number of events in the run is greater than 10,000 events, the IP profile is calculated every 10,000 events. Otherwise, the IP profile is calculated using all the events in the run. The interaction point in each event is obtained with the kinematical vertex fit using all the tracks associated with SVD hits. The position and the width in z direction of the IP profile are obtained by fitting the distributions of the primary vertices in the unit of event using the 3-dimensional Gaussian. Because the resolution of the vertex fit is greater than the beam size in x and y directions, the width in x and y direction of the IP profile is calculated from the beam size monitor. Typical width of the IP profile is $\sim 100\mu$ m, $\sim 5\mu$ m and ~ 3 mm in x, y and z direction, respectively.

The distribution of B meson decay points are expected to spread more widely than IP profile because the B mesons have the cms momentum of ~ 0.34 GeV. While this effect is negligible in z direction, in x and y direction the flight length of B mesons are greater than the IP profile. The nominal flight length of B mesons in x and y direction are estimated as 21 μ m from Monte Carlo simulation. Therefore, the IP profile is smeared by this amount in x and y directions.

4.5.2 Vertex Reconstruction of B_{CP}

The decay vertex of B_{CP} is reconstructed from the daughter charged π tracks that are associated with at least one SVD hit in r- ϕ plane and at least two SVD hits in the r-z plane. If neither one of two pions from $B^0 \to \pi^+\pi^$ candidate is associated with SVD hits, this event is not used in CP analysis. We use the same resolution function as that used for $B \to J/\psi K_S$ in the sin $2\phi_1$ and lifetime measurements [112]. It is confirmed that the difference of the vertex resolution between $B^0 \to \pi^+\pi^-$ decays and $B \to J/\psi K_S$ decays is small enough using a Monte Carlo simulation. Because of the IP profile constraint, it is possible to reconstruct the B_{CP} vertex even with a single track. The fraction of the single-track vertices is ~ 10%. For the multiple-track vertices, the quality of the vertex fit is further evaluated. We find using a Monte Carlo simulation that the vertex-fit χ^2 is correlated with the *B* decay length due to the tight IP constraint in the transverse plane. To avoid this correlation, we use the variable based on the *z* information only:

$$\xi_{z_{CP}} \equiv \frac{1}{2n} \sum_{k}^{n} \left(\frac{z_{k}^{\text{after}} - z_{k}^{\text{before}}}{\varepsilon_{k}^{\text{before}}} \right)^{2},$$

where n is the number of tracks used in the kinematical vertex fit⁸, $z_k^{\text{before}}(z_k^{\text{after}})$ is the position of the each track in z direction (at the closest approach to the origin) before (after) the vertex fit, and $\varepsilon_k^{\text{before}}$ is the error of the z_k^{before} . We require $\xi_{z_{CP}} < 100$ to eliminate poorly reconstructed vertices. We find that about 3% of the B_{CP} decay vertices are rejected in the data.

The event-by-event resolution function of the B_{CP} vertex point in the z direction, $R_{CP}^i(z_{CP})$ is defined using the measurement error of the vertex, $\sigma_{z_{CP}}^i$ and $\xi_{z_{CP}}^i$. The function form of $R_{CP}^i(\delta z_{CP})$ is defined as:

$$R_{CP}^{i}(\delta z_{CP}; \sigma_{z_{CP}}^{i}, \xi_{z_{CP}}^{i}) \equiv G(\delta z_{CP}; 0, (s_{CP}^{0} + s_{CP}^{1} \cdot \xi_{z_{CP}}^{i}) \cdot \sigma_{z_{CP}}^{i})$$
(4.18)

and

$$R^{i}_{CP}(\delta z_{CP}; \sigma^{i}_{z_{CP}}) \equiv G(\delta z_{CP}; 0, s^{\text{sigle}} \cdot \sigma^{i}_{z_{CP}})$$

$$(4.19)$$

for the multiple-track vertices and the single-track vertices, respectively, where δz_{CP} represents the residual of the measured z_{CP} from the true value. The parameters, s_{CP}^0 , s_{CP}^1 , f^{tail} and s^{single} , are determined by the *B* meson lifetime measurement using hadronic *B* meson decays⁹ in the real data [113] as listed in Table 4.11.

	parameters	or the resolution	<u> </u>
_	Parameters	Values	
	s^0_{CP}	$0.987 \ {}^+ \ {}^{0.117}_{0.124}$	
	s^1_{CP}	0.094 ± 0.008	

 0.972 ± 0.045

 s^{single}

Table 4.11: The parameters of the resolution of B_{CP} vertex.

4.5.3 Vertex Reconstruction of B_{tag}

The decay vertex of B_{tag} is determined inclusively from tracks not assigned to B_{CP} ; however, poorly reconstructed tracks (with a longitudinal position error in excess of 500 μ m, or without the associated SVD hits) as well as tracks that are likely to come from K_S decays (forming the K_S mass with another track, or more than 500 μ m away from the B_{CP} vertex in the r- ϕ plane) are not used. We repeat the vertex reconstruction by removing the track that gives the largest contribution to the reduced χ^2 (χ^2 /ndf) until the resulting χ^2 satisfies $\chi^2/\text{ndf} < 20$ or only one track is left. If, however, the track to be removed is a lepton with the cms momentum greater than 1.1 GeV/c, we keep the lepton and remove the track with the second largest contribution. This is because the high-momentum leptons are likely to come from primary semi-leptonic B decays. The presence of a secondary charm ($b \rightarrow c$) decay vertex in B_{tag} results in a shift of the reconstructed vertex point toward charm flight direction and degrades the vertex resolution (*Non-Primary tracks effect*). A Monte Carlo simulation study shows that the shift and the resolution of the B_{tag} decay vertex are ~ 20 μ m and ~ 140 μ m (rms), respectively, while the resolution of the fully reconstructed B decay vertex is ~ 75 μ m (rms).

The fraction of the single-track vertices for B_{tag} is ~ 22%. For the multiple-track vertices, $\xi_{z_{\text{tag}}}$ is defined in the same way as $\xi_{z_{CP}}$. We require $\xi_{z_{\text{tag}}} < 100$ to eliminate poorly reconstructed vertices. We find that about 1% of the B_{tag} decay vertices are rejected in the data.

The event-by-event resolution function of the B_{tag} vertex in the z direction due to the detector resolution, $R_{\text{tag,detector}}^{i}(z_{\text{tag}})$ is parameterized in the same way as that for the B_{CP} vertex:

$$R_{\delta \text{tag,detctor}}^{i}(z_{\text{tag}}; \sigma^{i}_{z_{\text{tag}}}, \xi^{i}_{z_{\text{tag}}}) \equiv G(\delta z_{\text{tag}}; 0, (s^{0}_{\text{tag}} + s^{1}_{\text{tag}} \cdot \xi^{i}_{z_{\text{tag}}}) \cdot \sigma^{i}_{z_{\text{tag}}})$$
(4.20)

and

$$R_{\text{tag,detctor}}^{i}(\delta z_{\text{tag}}; \sigma_{z_{\text{tag}}}^{i}) \equiv G(\delta z_{\text{tag}}; 0, s^{\text{sigle}} \cdot \sigma_{z_{\text{tag}}}^{i})$$
(4.21)

⁸For single-track vertices ξ cannot be defined.

 ${}^{9}B^{0} \rightarrow J/\psi K_{S}, B^{0} \rightarrow J/\psi K^{*0}(K^{*0} \rightarrow K^{-}\pi^{+}), B^{0} \rightarrow D^{(*)-}\pi^{+}, B^{0} \rightarrow D^{*-}\rho^{+}, B^{+} \rightarrow J/\psi K^{+} \text{ and } B^{+} \rightarrow \overline{D}{}^{0}\pi^{+}$

for the multiple-track vertices and the single-track vertices, respectively, where $\delta_{\mathcal{X}_{\text{tag}}}$ represents the difference of the measured z_{tag} from the true value. $\sigma_{z_{\text{tag}}}^{i}$ is the measurement error of z_{tag} . The parameters, s_{tag}^{0} , s_{tag}^{1} , f^{tail} and s^{single} , are determined by the *B* meson lifetime measurement using hadronic *B* meson decays as listed in Table 4.12. s^{single} is common to the B_{CP} vertex.

The resolution due to the non-primary tracks effect: $R_{\rm NP}^i(\delta' z_{\rm tag})$ is studied in Monte Carlo by comparing the generic Monte Carlo and the special Monte Carlo in which all secondary particles are forced to decay with zero lifetimes at *B* meson decay points. We assume $R_{\rm NP}^i(z_{\rm tag})$ consists of the prompt component, expressed by the Dirac's δ function, and the components which account for the smearing due to the lifetime functions defined as:

$$E_{\rm p}(x,\tau) \equiv \begin{cases} (1/\tau) \cdot \exp(-x/\tau) & (x>0) \\ 0 & (x\le 0) \end{cases},$$
(4.22)

$$E_{\rm n}(x,\tau) \equiv \begin{cases} 0 & (x>0) \\ (1/\tau) \cdot \exp(+x/\tau) & (x \le 0) \end{cases}.$$
(4.23)

Thus, $R_{\rm NP}^i(\delta' z_{\rm tag})$ is parameterized as:

$$R_{\rm NP}^{i}(\delta' z_{\rm tag}; \sigma_{z_{\rm tag}}^{i}, \xi_{z_{\rm tag}}) \equiv f_{\delta}^{\rm NP} \cdot \delta(\delta' z_{\rm tag}) + (1 - f_{\delta}^{\rm NP})[f_{\rm p}^{\rm NP} \cdot E_{\rm p}(\delta' z_{\rm tag}; \tau_{\rm p}^{\rm NP,i}) + (1 - f_{\rm p}^{\rm NP}) \cdot E_{\rm n}(\delta' z_{\rm tag}; \tau_{\rm n}^{\rm NP,i})],$$

$$(4.24)$$

where $\delta' z_{\text{tag}}$ represents the difference of z_{tag} due to non-primary tracks effect, and f_{δ}^{NP} and f_{p}^{NP} are the fraction of the prompt component and the positive part in the lifetime component, respectively. f_{p}^{NP} is determined by the Monte Carlo sample while f_{δ}^{NP} is obtained by the B^{0} meson lifetime measurement using the real data. Moreover, the lifetime parameters $\tau_{p}^{\text{NP},i}$ and $\tau_{n}^{\text{NP},i}$ are defined in event by event as

$$\tau_{\rm p}^{\rm NP,i} \equiv \tau_{\rm p}^{\rm NP,0} + \tau_{\rm p}^{\rm NP,1} \cdot (s_{\rm tag}^0 + s_{\rm tag}^1 \cdot \xi_{z_{\rm tag}}^i) \cdot \sigma_{z_{\rm tag}}^i / (\beta\gamma)_{\Upsilon(4{\rm S})}$$

$$\tag{4.25}$$

$$\tau_{\rm n}^{\rm NP,i} \equiv \tau_{\rm n}^{\rm NP,0} + \tau_{\rm n}^{\rm NP,1} \cdot (s_{\rm tag}^0 + s_{\rm tag}^1 \cdot \xi_{z_{\rm tag}}^i) \cdot \sigma_{z_{\rm tag}}^i / (\beta\gamma)_{\Upsilon(4{\rm S})}$$

$$\tag{4.26}$$

for the case that both pion tracks satisfy the SVD hits requirement and

$$\tau_{\rm p}^{\rm NP,i} \equiv \tau_{\rm p}^{\rm NP,0} + \tau_{\rm p}^{\rm NP,1} \cdot s^{\rm sigle} \cdot \sigma_{z_{\rm tag}}^{i} / (\beta\gamma)_{\Upsilon(4\rm S)}$$

$$\tag{4.27}$$

$$\tau_{\rm n}^{\rm NP,i} \equiv \tau_{\rm n}^{\rm NP,0} + \tau_{\rm n}^{\rm NP,1} \cdot s^{\rm sigle} \cdot \sigma_{z_{\rm tag}}^{i} / (\beta\gamma)_{\Upsilon(4\rm S)}$$

$$\tag{4.28}$$

for the case that only one track satisfies the SVD hits requirement, where the parameters $\tau_p^{NP,0}$, $\tau_p^{NP,1}$, $\tau_n^{NP,0}$ and $\tau_n^{NP,1}$, which are listed in Table 4.12, are determined by Monte Carlo.

The overall resolution function of B_{tag} vertex is defined as

$$R_{\text{tag}}{}^{i}(\delta z_{\text{tag}};\sigma_{z_{\text{tag}}}^{i},\xi_{z_{\text{tag}}}^{i}) \equiv R_{\text{tag,detctor}}{}^{i}(\delta z_{\text{tag}};\sigma_{z_{\text{tag}}}^{i},\xi_{z_{\text{tag}}}^{i}) \otimes R_{\text{NP}}{}^{i}(\delta z_{\text{tag}};\sigma_{z_{\text{tag}}}^{i},\xi_{z_{\text{tag}}}^{i})$$
(4.29)

where the operator \otimes expresses the convolution, i.e.

$$f(x) \otimes g(x) \equiv \int_{-\infty}^{\infty} dx' f(x') \cdot g(x - x').$$
(4.30)

Table 4.12: The parameter	rs of the reso	olution of B	$\mathcal{S}_{\mathrm{tag}}$ vertex.
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Parameters	Values		
	multiple tracks single track		
$s_{ m tag}^0$	0.778 ± 0.048		
$s_{ m tag}^1$	0.044 ± 0.002		
$f^{ m NP}_{\delta}$	$0.555 \ {}^{+}_{-} \ {}^{0.041}_{0.043}$	$0.701 \ {}^{+}_{-} \ {}^{0.039}_{0.042}$	
$f_{ m p}^{ m NP}$	0.955 ± 0.004	$0.790 \ {}^+_{-} \ {}^{0.020}_{0.021}$	
$ au_{ m p}^{ m NP,0}$	$-0.010 \pm 0.011 \ \rm ps$	$0.108 \stackrel{+}{_{-}} \stackrel{0.068}{_{-}} \mathrm{ps}$	
$ au_{ m p}^{ m NP,1}$	$0.927 \ {}^+_{-} \ {}^{0.025}_{0.024}$	$1.321 \ {}^+_{- \ 0.094} \ {}^{0.099}_{- \ 0.094}$	
$ au_{ m n}^{ m NP,0}$	$-0.194 \stackrel{+}{_{-}} \stackrel{0.078}{_{-}} \mathrm{ps}$	$-0.281 \stackrel{+}{_{-}} \stackrel{0.130}{_{-}} \text{ps}$	
$ au_{ m n}^{ m NP,1}$	$1.990 \stackrel{+}{_{-}} \stackrel{0.182}{_{-}} \stackrel{0.182}{_{0.169}}$	$1.583 \stackrel{+}{_{-}} \stackrel{0.213}{_{-}} \stackrel{0.184}{_{-}}$	

Chapter 5

Determination of CP Asymmetry

The time-dependent CP asymmetry parameters, $\mathcal{A}_{\pi\pi}$ and $\mathcal{S}_{\pi\pi}$ are determined from the Δt distribution for 760 $B^0 \to \pi^+\pi^-$ candidate events using an unbinned maximum likelihood method. We estimate the statistical significance with the *Frequentist Approach* established by J. Feldman and D. Cousins [114] which is described in Section 6.1. We calculate the confidence region within the physical boundary and perform the hypothesis tests for the null CP asymmetry.

5.1 Unbinned Maximum likelihood fit method

We obtain a set of N = 760 independently measured values Δt_i , where the subscript *i* indecats call of $B^0 \to \pi^+ \pi^$ candidates. We define the normalized probability density $P(\Delta t; \mathcal{A}_{\pi\pi}, \mathcal{S}_{\pi\pi})$ for Δt as a function of $\mathcal{A}_{\pi\pi}$ and $\mathcal{S}_{\pi\pi}$. We construct the likelihood for the total N measurements as:

$$\mathcal{L}(\mathcal{A}_{\pi\pi}, \mathcal{S}_{\pi\pi}) = \prod_{i}^{N} P(\Delta t_{i}; \mathcal{A}_{\pi\pi}, \mathcal{S}_{\pi\pi}).$$
(5.1)

In the analysis, we obtained the most probable values of $\mathcal{A}_{\pi\pi}$ and $\mathcal{S}_{\pi\pi}$ by minimizing

$$-2\ln \mathcal{L}(\mathcal{A}_{\pi\pi}, \mathcal{S}_{\pi\pi}) = -2\sum_{i}^{N} \ln \left[P(\Delta t_i; \mathcal{A}_{\pi\pi}, \mathcal{S}_{\pi\pi}) \right]$$
(5.2)

with MINUIT [115], which is a part of the numerical computing package distributed by CERN.

Since the quantity $-2 \ln \mathcal{L}(\mathcal{A}_{\pi\pi}, \mathcal{S}_{\pi\pi})$ behaves like χ^2 in the large sample limit, the 1-standard-deviation positive (negative) statistical errors, $\sigma_{\mathcal{A}_{\pi\pi}}$ and $\sigma_{\mathcal{S}_{\pi\pi}}$ as:

$$-2\ln \mathcal{L}(\widehat{\mathcal{A}_{\pi\pi}} + \sigma_{\mathcal{A}_{\pi\pi}}, \widehat{\mathcal{S}_{\pi\pi}} + \sigma \mathcal{S}_{\pi\pi}) = -2\ln \mathcal{L}(\widehat{\mathcal{A}_{\pi\pi}}, \widehat{\mathcal{S}_{\pi\pi}}) + 1,$$
(5.3)

where $\widehat{\mathcal{A}_{\pi\pi}}$ and $\widehat{\mathcal{S}_{\pi\pi}}$ are the central values of $\mathcal{A}_{\pi\pi}$ and $\mathcal{S}_{\pi\pi}$, respectively. $P(\Delta t_i; \mathcal{A}_{\pi\pi}, \mathcal{S}_{\pi\pi})$ depends on the measurment error of the vertex, the quality of the flavor tagging and other quantities reflecting on the response of the detector. The detail of the probability density is explained in Section 5.2.

5.2 Probability Density Function for Proper Time Difference

 $P^{i}(\Delta t; \mathcal{A}_{\pi\pi}, \mathcal{S}_{\pi\pi})$, is defined as the linear combination of four components, the $B^{0} \to \pi^{+}\pi^{-}$ signal $P^{i}_{\pi\pi}(\Delta t; \mathcal{A}_{\pi\pi}, \mathcal{S}_{\pi\pi})$, the $B^{0} \to K^{+}\pi^{-}$ background $P^{i}_{K\pi}(\Delta t)$, the continuum background shape $P^{i}_{q\bar{q}}(\Delta t)$, and the *outlier component* $P_{\mathrm{ol}}(\Delta t)$ as:

$$P^{i}(\Delta t; \mathcal{A}_{\pi\pi}, \mathcal{S}_{\pi\pi}) = (1 - f_{\text{Ol}}) \cdot \left[f^{i}_{\pi\pi} \cdot P^{i}_{\pi\pi}(\Delta t; \mathcal{A}_{\pi\pi}, \mathcal{S}_{\pi\pi}) \otimes R^{i}_{\text{sig}}(\Delta t) + f^{i}_{K\pi} \cdot P^{i}_{K\pi}(\Delta t) \otimes R^{i}_{\text{sig}}(\Delta t) + f^{i}_{q\bar{q}} \cdot P^{i}_{q\bar{q}}(\Delta t) \right] + f_{\text{Ol}} \cdot P_{\text{Ol}}(\Delta t),$$
(5.4)

where $R^i_{sig}(\Delta t)$ is a resolution function of Δt for the $B^0 \to \pi^+\pi^-$ and the $B^0 \to K^+\pi^-$ events and f_{ol} is the fraction of the outlier component while $f^i_{\pi\pi}$, $f^i_{K\pi}$ and $f^i_{q\bar{q}}$ are the fractions of $B^0 \to \pi^+\pi^-$ signal, $B^0 \to K^+\pi^-$ background and continuum background, respectively. The $f^i_{\pi\pi}$, $f^i_{K\pi}$ and $f^i_{q\bar{q}}$ are defined event by event and normalized as:

$$f_{\pi\pi}^{i} + f_{K\pi}^{i} + f_{q\bar{q}}^{i} = 1$$
(5.5)

Because the event topology of $B^0 \to \pi^+\pi^-$ decays and that of $B^0 \to K^+\pi^-$ decays are nearly identical, we use the common resolution function for both components. The outlier component, which is discussed in Section 5.2.2, expresses a very long tail in the Δt distribution, which cannot be expressed by the detector resolution.

5.2.1 Probability Density Function for signal

The probability density function of the $B^0 \rightarrow \pi^+\pi^-$ signal for the *i*-th candidate is given by

$$P_{\pi\pi}^{i}(\Delta t; q, w_{\text{tag}}^{i}; \mathcal{A}_{\pi\pi}, \mathcal{S}_{\pi\pi}) = \frac{\exp(-|\Delta t|/\tau_{B^{0}})}{4\tau_{B^{0}}} \bigg\{ 1 + q \cdot (1 - 2w_{\text{tag}}^{l}) \bigg[\mathcal{A}_{\pi\pi} \cdot \cos(\Delta m_{d} \cdot \Delta t) + \mathcal{S}_{\pi\pi} \cdot \sin(\Delta m_{d} \cdot \Delta t) \bigg] \bigg\}$$
(5.6)

where q is the flavor of tagging B meson obtained and w_{tag}^l is the wrong tag fraction for the tagging quality region to which the candidate belongs. The lifetime of B^0 mesons (τ_{B^0}) and the $B^0 - \overline{B}^0$ mixing parameter (Δm_d) are fixed to the world averages [42]:

$$\tau_{B^0} = 1.542 \pm 0.016 \text{ ps}$$
 (5.7)

and

$$\Delta m_d = 0.489 \pm 0.008 \ \hbar \ \mathrm{ps}^{-1}.$$
(5.8)

5.2.2 Resolution function

The resolution function of Δt for the $B^0 \to \pi^+ \pi^-$ signal $R^i_{sig}(\delta \Delta t)$ consists of the detector resolution of the vertex measurement $R^i_{det}(\delta \Delta z)$, and the correction for the kinematical approximation that the *B* mesons are at rest in the cms $R^i_{kine}(\delta \Delta t)$, where $\delta \Delta t$ and $\delta \Delta z$ represent the differences of the measured value and the true value for Δt and Δz , respectively [113]. Thus,

$$R^{i}_{\rm sig}(\delta\Delta t) \equiv R^{i}_{\rm det}(\delta\Delta t) \otimes R^{i}_{\rm kine}(\delta\Delta t), \tag{5.9}$$

where $R_{\text{det}}^i(\delta\Delta t) \equiv R_{\text{det}}^i(\delta\Delta z)/(\beta\gamma)_{\Upsilon(4S)}c$.

The detector resolution $R_{det}^i(\delta \Delta z)$ is defined as a convolution of $R_{CP}^i(\delta z_{CP})$ and $R_{tag}^i(\delta z_{tag})$, which are described in Sections 4.5.2 and 4.5.3:

$$R^{i}_{det}(\delta\Delta z) \equiv R^{i}_{CP}(\delta z_{CP}) \circ R_{tag}{}^{i}(-\delta z_{tag}), \qquad (5.10)$$

where the minus sign is caused that Δz is defined as $z_{CP} - z_{\text{tag}}$. Here the operator \circ expresses a succession of two operation.¹ Figure 5.1 shows the average detector resolution function of the 760 $B^0 \rightarrow \pi^+\pi^-$ candidate. The Δt resolution due to the detector resolution of the vertex measurement is ~ 1.4 ps (RMS).

The correction for the kinematical approximation $R^i_{\text{kine}}(\delta \Delta t)$ is calculated analytically as a function of the E^*_B and θ^*_B from the kinematics of the $\Upsilon(4S)$ two-body decay, where E^*_B and θ^*_B are the energy and polar angle of the reconstructed B_{CP} candidate in the cms. The residual of the measured Δt defined in Equation 4.17 from the true value, Δt_{true} that is defined in Equation 4.16 can be given as:

$$\delta\Delta t \equiv \Delta t - \Delta t_{\rm true} = \frac{z_{CP} - z_{\rm tag}}{(\beta\gamma)_{\Upsilon(4S)} \cdot c} - (t_{CP} - t_{\rm tag})$$

$$= \frac{t_{CP}c \cdot (\beta\gamma)_{CP} - t_{\rm tag}(\beta\gamma)_{\rm tag}c \cdot}{(\beta\gamma)_{\Upsilon(4S)} \cdot c} - (t_{CP} - t_{\rm tag})$$

$$= \left(\frac{(\beta\gamma)_{CP}}{(\beta\gamma)_{\Upsilon(4S)}} - 1\right) \cdot t_{CP} - \left(\frac{(\beta\gamma)_{\rm tag}}{(\beta\gamma)_{\Upsilon(4S)}} - 1\right) \cdot t_{\rm tag}, \tag{5.11}$$

where $(\beta\gamma)_{CP}$ and $(\beta\gamma)_{tag}$ are the Lorentz boost factors of B_{CP} and B_{tag} , respectively. Moreover, $(\beta\gamma)_{CP}$ and $(\beta\gamma)_{tag}$ are expressed as

$$(\beta\gamma)_{CP} = (\beta\gamma)_{\Upsilon(4S)} \left(\frac{E_B^*}{m_B} + \frac{|\boldsymbol{p}_B^*|\cos\theta_B^*}{\beta_{\Upsilon(4S)}m_B} \right)$$
(5.12)

and

$$(\beta\gamma)_{\text{tag}} = (\beta\gamma)_{\Upsilon(4S)} \left(\frac{E_B^*}{m_B} - \frac{|\boldsymbol{p}_B^*| \cos \theta_B^*}{\beta_{\Upsilon(4S)} m_B} \right),$$
(5.13)

¹i.e. $f(x,y) \otimes (g(x) \circ h(y)) = [f(x,y) \otimes g(x)] \otimes h(y).$

where m_B is the mass of B^0 meson, $|\mathbf{p}_B^*| = 0.34 \text{ GeV}/c$ is the cms momentum of B_{CP} , and $\beta_{\Upsilon(4S)} = 0.391$ is the velocity of $\Upsilon(4S)$ in the unit of c. Because t_{CP} and t_{tag} distributions are given by the $E_p(t_{CP}; \tau_{B^0})$ and $E_p(t_{\text{tag}}; \tau_{B^0})$, respectively, the probability density to obtain $\delta \Delta t$ and Δt_{true} simultaneously is given by²:

$$P(\delta\Delta t, \Delta t_{\rm true}) = \int_0^\infty dt_{CP} \int_0^\infty dt_{\rm tag} E_{\rm p}(t_{CP}; \tau_{B^0}) E_{\rm p}(t_{\rm tag}; \tau_{B^0}) \delta(\Delta t_{\rm true} - (t_{CP} - t_{\rm tag})) \\ \times \delta(\delta\Delta t - \{ [(\beta\gamma)_{CP}/(\beta\gamma)_{\Upsilon(4{\rm S})} - 1] \cdot t_{CP} - [(\beta\gamma)_{\rm tag}/(\beta\gamma)_{\Upsilon(4{\rm S})} - 1] \cdot t_{\rm tag} \}), \quad (5.14)$$

and the probability density to obtain Δt_{true} is given by:

$$P(\Delta t_{\rm true}) = \int_{0}^{\infty} dt_{CP} \int_{0}^{\infty} dt_{\rm tag} E_{\rm p}(t_{CP};\tau_{B^{0}}) E_{\rm p}(t_{\rm tag};\tau_{B^{0}}) \delta(\Delta t_{\rm true} - (t_{CP} - t_{\rm tag})).$$
(5.15)

 $R^i_{\text{kine}}(\delta\Delta t)$ is derived from $R^i_{\text{kine}}(\delta\Delta t) = P(\delta\Delta t, \Delta t_{\text{true}})/P(\Delta t_{\text{true}})$ as:

$$R_{\rm kine}^{i}(\delta\Delta t) = \begin{cases} E_{\rm p}\left(\delta\Delta t - \left[\left(\frac{E_{B}^{*}}{m_{B}} - 1\right)\Delta t_{\rm true} + \frac{|\boldsymbol{p}_{B}^{*}|\cos\theta_{B}^{*}|}{\beta_{\Upsilon(4S)}m_{B}}|\Delta t_{\rm true}|\right]; \left|\frac{|\boldsymbol{p}_{B}^{*}|\cos\theta_{B}^{*}|}{\beta_{\Upsilon(4S)}m_{B}}|\tau_{B^{0}}\right) & (\cos\theta_{B}^{*} > 0) \\ \delta(\delta\Delta t - \left(\frac{E_{B}^{*}}{m_{B}} - 1\right)\Delta t_{\rm true}) & (\cos\theta_{B}^{*} = 0) (5.16) \\ E_{\rm n}\left(\delta\Delta t - \left[\left(\frac{E_{B}^{*}}{m_{B}} - 1\right)\Delta t_{\rm true} + \frac{|\boldsymbol{p}_{B}^{*}|\cos\theta_{B}^{*}|}{\beta_{\Upsilon(4S)}m_{B}}|\Delta t_{\rm true}|\right]; \left|\frac{|\boldsymbol{p}_{B}^{*}|\cos\theta_{B}^{*}|}{\beta_{\Upsilon(4S)}m_{B}}|\tau_{B^{0}}\right) & (\cos\theta_{B}^{*} < 0) \end{cases}$$

The signal probability density function with the kinematical correction is calculated as:

$$P_{\pi\pi}^{i}(\Delta t; q, w_{\text{tag}}^{i}; \mathcal{A}_{\pi\pi}, \mathcal{S}_{\pi\pi}) \otimes R_{\text{kine}}^{i}(\Delta t)$$

$$= \left(1 \pm \frac{|\boldsymbol{p}_{B}^{*}| \cos \theta_{B}^{*}}{\beta_{\Upsilon(4S)} E_{B}^{*}}\right) \cdot \frac{\exp(-|\Delta t|/\tau_{B^{0}}^{\prime})}{4\tau_{B^{0}}^{\prime}}$$

$$\times \left\{1 + \frac{q \cdot (1 - 2w_{\text{tag}}^{l})}{1 + (x_{d}^{\prime})^{2}} \left[(\mathcal{A}_{\pi\pi} + x_{d}^{\prime} \mathcal{S}_{\pi\pi}) \cdot \cos(\Delta m_{d}^{\prime} \cdot \Delta t) + (\mathcal{S}_{\pi\pi} - x_{d}^{\prime} \mathcal{A}_{\pi\pi}) \cdot \sin(\Delta m_{d}^{\prime} \cdot \Delta t)\right]\right\}, \quad (5.17)$$

where

$$\tau_{B^0}' = \tau_{B^0} \cdot (E_B^* / m_B \pm |\mathbf{p}_B^*| \cos \theta_B^* / \beta_{\Upsilon(4S)} m_B), \tag{5.18}$$

$$\Delta m'_d = \Delta m_d / (E^*_B / \mathbf{m}_B \pm |\mathbf{p}^*_B| \cos \theta^*_B / \beta_{\Upsilon(4S)} m_B), \qquad (5.19)$$

$$x'_d = (|\boldsymbol{p}^*_B| \cos\theta^*_B / \beta_{\Upsilon(4S)} E^*_B) \cdot \Delta m_d \tau_{B^0}.$$
(5.20)

Here, in the duplicated signs in the above equations, + is for $\Delta t \ge 0$ and - is for $\Delta t < 0$.

There is a long tail in the Δt distribution, which cannot be expressed by the detector resolution. This outlier component, $P_{\rm ol}(\Delta t)$, is represented by a single Gaussian with zero mean and the event-independent width, $\sigma_{\rm ol}$, as:

$$P_{\rm ol}(\Delta t) \equiv G(\Delta t; 0, \sigma_{\rm ol}). \tag{5.21}$$

The fraction of outlier component, f_{ol} , and σ_{ol} , are determined from the B^0 lifetime measurement in the real data as listed in Table 5.1. Different values are used for f_{ol} depending on whether both vertices are reconstructed with multiple tracks or not.

Table 5.1: The parameters of the outliers.				
Parameters	Values			
	multiple tracks	single track		
$\sigma_{ m ol}$	$42.0 + \frac{4.6}{-3.5}$ ps			
$f_{\rm ol}$	$(1.65 \ ^+_{-0.82}) \times 10^{-4}$	$0.0269 \ {}^+_{-} \ {}^{0.0019}_{0.0018}$		

5.2.3 Treatment of $B^0 \to K^+\pi^-$ Background

The probability density function for the $B^0 \to K^+\pi^-$ background is described as:

$$P_{K\pi}^{i}(\Delta t; q, w_{\text{tag}}^{l}; \mathcal{A}_{K\pi}) = \frac{\exp(-|\Delta t|/\tau_{B^{0}})}{4\tau_{B^{0}}} \left[1 + q \cdot (1 - 2w_{\text{tag}}^{l})\mathcal{A}_{K\pi} \cdot \cos(\Delta m_{d} \cdot \Delta t)\right].$$
(5.22)

In the nominal fit, we assume that there is no CP asymmetry for the $B^0 \to K^+\pi^-$ mode, i.e. $\mathcal{A}_{K\pi}$ is fixed to be zero. The effect of the non-zero $\mathcal{A}_{K\pi}$ possibility is included in the systematic uncertainty.

²The definition of $E_{\rm p}(t;\tau)$ is Equation 4.22.



Figure 5.1: The average detector resolution of the 760 $B^0 \to \pi^+\pi^-$ candidate.

5.2.4 Probability Density Function for Continuum Background

The probability density function for the continuum background is determined using the ΔE - $M_{\rm bc}$ sideband region. It is modeled to be a linear combination of the lifetime function and the delta function, smeared by the sum of two Gaussians as:

$$P_{q\bar{q}}^{i}(\Delta t;q) \equiv \frac{1+q\cdot\delta_{q\bar{q}}}{2} \times \left\{ (1-f_{\delta}^{q\bar{q}})\frac{\exp(-|\Delta t|/\tau_{q\bar{q}})}{2\tau_{q\bar{q}}} \otimes \left[(1-f_{\text{tail}}^{q\bar{q}})\cdot G(\Delta t;\mu_{\tau}^{q\bar{q}},\sigma_{\text{main}}^{q\bar{q}}) + f_{\text{tail}}^{q\bar{q}}\cdot G(\Delta t;\mu_{\tau}^{q\bar{q}},\sigma_{\text{tail}}^{q\bar{q}}) \right]$$

$$+f_{\delta}^{q\bar{q}}\delta(\Delta t)\otimes\left[\left(1-f_{\mathrm{tail}}^{q\bar{q}}\right)\cdot G(\Delta t;\mu_{\delta}^{q\bar{q}},\sigma_{\mathrm{main}}^{q\bar{q}})+f_{\mathrm{tail}}^{q\bar{q}}\cdot G(\Delta t;\mu_{\delta}^{q\bar{q}},\sigma_{\mathrm{tail}}^{q\bar{q}})\right]\bigg\},\tag{5.23}$$

$$\sigma_{\min}^{q\bar{q}} \equiv s_{\min}^{q\bar{q}} \cdot \sigma_{\Delta t}^{i}$$
(5.24)

$$\sigma_{\text{tail}}^{qq} \equiv s_{\text{tail}}^{qq} \cdot s_{\text{main}}^{qq} \cdot \sigma_{\Delta t}^{i}$$
(5.25)

where $\sigma_{\Delta t}^{i}$ is the measurement error of Δt , and $\delta_{q\bar{q}}$ is the CP asymmetry of the background. The parameters, $f_{\delta}^{q\bar{q}}$, $\tau_{q\bar{q}}, \mu_{q\bar{q}}^{q\bar{q}}, \mu_{\delta}^{q\bar{q}}, f_{tail}^{q\bar{q}}$, $s_{main}^{q\bar{q}}$ and $s_{tail}^{q\bar{q}}$ are determined by the fit to the sideband data of $(5.2 < M_{bc} < 5.26, -0.3 < \Delta E < 0.5)$ and $(5.26 < M_{bc}, 0.1 < \Delta E < 0.5)$. In the nominal fit, $\delta_{q\bar{q}}$ is fixed to be zero and the difference of the number of events observed between q = +1 and q = -1 in the sideband region is included by the systematic uncertainty. The other parameters are determined depending on whether both vertices are reconstructed with multiple tracks or not. If one of the vertices is reconstructed with a single track, $f_{\delta}^{q\bar{q}}$ is fixed to be 1. Figure 5.2 shows the Δt distributions for the sideband region and the fitted probability density functions are superimposed. The obtained probability density function represents the background shape well.

Table 5.2: The Δt shape parameters for the continuum background which are determined using the sideband data. Parameters Values

Parameters	Values		
	multiple tracks	single track	
$f^{q \bar{q}}_{\delta}$	$0.9857 \ {}^{+}_{-} \ {}^{0.0041}_{0.0055}$	1 (fixed)	
$ au_{qar q}$	$2.37 \ {}^{+}_{-} \ {}^{0.44}_{0.34}$		
$\mu^{qar{q}}_{ au}$	$0.45 \ ^+_{-} \ ^{0.40}_{0.34}$		
$\mu^{qar{q}}_{\delta}$	-0.0547 ± 0.0055	-0.045 ± 0.013	
$f_{ ext{tail}}^{qar{q}}$	$0.179 \ {}^+_{-} \ {}^{0.019}_{0.017}$	$0.160 \ ^+_{-} \ ^{0.021}_{0.019}$	
$s_{ m main}^{qar{q}}$	$1.093 \ ^+_{-} \ ^{0.016}_{0.017}$	1.042 ± 0.021	
$s^{qar{q}}_{ ext{tail}}$	$2.228 \ {}^+ \ {}^{0.077}_{0.075}$	$2.53 \ ^+_{-} \ ^{0.12}_{0.11}$	



Figure 5.2: The Δt distributions for the sideband region. The solid lines show the fit results.

5.2.5 Signal Probability

The probabilities that an event is the $B^0 \to \pi^+\pi^-$ signal $f^i_{\pi\pi}$, the $B^0 \to K^+\pi^-$ background $f^i_{K\pi}$, and the continuum background $f^i_{q\bar{q}}$ are defined as a function of $M_{\rm bc}$ and ΔE , , and depend on the \mathcal{LR} -r region (Table 4.4).

$$f^{i}_{\pi\pi}(M_{\rm bc},\Delta E,l) \equiv \frac{F_{\pi\pi}(M_{\rm bc},\Delta E) \cdot g^{l}_{\pi\pi}}{F_{\pi\pi}(M_{\rm bc},\Delta E) \cdot g^{l}_{\pi\pi} + F_{K\pi}(M_{\rm bc},\Delta E) \cdot g^{l}_{K\pi} + F_{q\bar{q}}(M_{\rm bc},\Delta E) \cdot g^{l}_{q\bar{q}}},$$
(5.26)

$$f_{K\pi}^{i}(M_{\rm bc},\Delta E,l) \equiv \frac{F_{K\pi}(M_{\rm bc},\Delta E) \cdot g_{K\pi}^{l}}{F_{\pi\pi}(M_{\rm bc},\Delta E) \cdot g_{\pi\pi}^{l} + F_{K\pi}(M_{\rm bc},\Delta E) \cdot g_{K\pi}^{l} + F_{q\bar{q}}(M_{\rm bc},\Delta E) \cdot g_{q\bar{q}}^{l}}$$
(5.27)

and

$$f_{q\bar{q}}^{i}(M_{\rm bc},\Delta E,l) \equiv \frac{F_{q\bar{q}}(M_{\rm bc},\Delta E) \cdot g_{q\bar{q}}^{l}}{F_{\pi\pi}(M_{\rm bc},\Delta E)g_{\pi\pi}^{l} + F_{K\pi}(M_{\rm bc},\Delta E) \cdot g_{K\pi}^{l} + F_{q\bar{q}}(M_{\rm bc},\Delta E) \cdot g_{q\bar{q}}^{l}},$$
(5.28)

where *l* indicates that the event belongs to the *l*-th bin of 12 \mathcal{LR} -*r* regions, and $g_{\pi\pi}^l$, $g_{K\pi}^l$ and $g_{\pi\pi}^l$ represent the S/N ratio in each \mathcal{LR} -*r* region, while $F_{\pi\pi}(M_{\rm bc}, \Delta E)$, $F_{K\pi}(M_{\rm bc}, \Delta E)$ and $F_{q\bar{q}}(M_{\rm bc}, \Delta E)$ represent the $M_{\rm bc}$ and ΔE dependence in each component. $F_{\pi\pi}(M_{\rm bc}, \Delta E)$, $F_{K\pi}(M_{\rm bc}, \Delta E)$ and $F_{q\bar{q}}(M_{\rm bc}, \Delta E)$ are defined as:

$$F_{\pi\pi}(M_{\rm bc}, \Delta E) \equiv P_{M_{\rm bc}}^{B\overline{B}}(M_{\rm bc}) P_{\Delta E}^{\pi\pi}(\Delta E) / \iint_{\substack{\text{signal}\\ \text{region}}} d(M_{\rm bc}) d(\Delta E) P_{M_{\rm bc}}^{B\overline{B}}(M_{\rm bc}) P_{\Delta E}^{\pi\pi}(\Delta E),$$
(5.29)

$$F_{\pi\pi}(M_{\rm bc}, \Delta E) \equiv P_{M_{\rm bc}}^{B\overline{B}}(M_{\rm bc}) P_{\Delta E}^{K\pi}(\Delta E) / \iint_{\substack{\text{signal}\\\text{region}}} d(M_{\rm bc}) d(\Delta E) P_{M_{\rm bc}}^{B\overline{B}}(M_{\rm bc}) P_{\Delta E}^{K\pi}(\Delta E)$$
(5.30)

and

$$F_{q\bar{q}}(M_{\rm bc},\Delta E) \equiv P_{M_{\rm bc}}^{q\bar{q}}(M_{\rm bc};\alpha)P_{\Delta E}^{q\bar{q}}(\Delta E;c_1) / \iint_{\substack{\text{signal}\\\text{region}}} d(M_{\rm bc})d(\Delta E) P_{M_{\rm bc}}^{q\bar{q}}(M_{\rm bc};\alpha)P_{\Delta E}^{q\bar{q}}(\Delta E;c_1), \quad (5.31)$$

where $P_{M_{\rm bc}}^{B\overline{B}}(M_{\rm bc})$, $P_{M_{\rm bc}}^{q\bar{q}}(M_{\rm bc};\alpha)$, $P_{\Delta E}^{\pi\pi}(\Delta E)$, $P_{\Delta E}^{K\pi}(\Delta E)$ and $P_{\Delta E}^{q\bar{q}}(\Delta E;c_1)$ are defined in Section 4.3.3. Because we find c_1 , which is the slope parameter of the ΔE distribution of continuum background, depends on l, c_1 is varied bin-by-bin as shown in Table 5.3.

The fractions, $g_{\pi\pi}^l$, $g_{K\pi}^l$ and $g_{\pi\pi}^l$, are defined as:

$$g_{\pi\pi}^{l} \equiv \frac{n_{\pi\pi} \cdot f(l;\pi\pi)}{n_{\pi\pi} \cdot f(l;\pi\pi) + n_{K\pi} \cdot f(l;K\pi) + n_{q\bar{q}} \cdot f(l;q\bar{q})},$$
(5.32)

$$g_{K\pi}^{l} \equiv \frac{n_{K\pi} \cdot f(l; K\pi)}{n_{\pi\pi} \cdot f(l; \pi\pi) + n_{K\pi} \cdot f(l; K\pi) + n_{q\bar{q}} \cdot f(l; q\bar{q})}$$
(5.33)

and

g

$${}^{l}_{q\bar{q}} \equiv \frac{n_{q\bar{q}} \cdot f(l;q\bar{q})}{n_{\pi\pi} \cdot f(l;\pi\pi) + n_{K\pi} \cdot f(l;K\pi) + n_{q\bar{q}} \cdot f(l;q\bar{q})},$$
(5.34)

where $f(l; \pi\pi)$, $f(l; K\pi)$ and $f(l; q\bar{q})$ are the fractions in each \mathcal{LR} -r regions for the $B^0 \to \pi^+\pi^-$, $B^0 \to K^+\pi^$ and continuum background, respectively. The normalization factors for each component, $n_{\pi\pi}$, $n_{K\pi}$ and $n_{q\bar{q}}$, are obtained from the S/N ratio.

 $f(l;\pi\pi)$, $f(l;K\pi)$ and $f(l;q\bar{q})$ satisfy:

$$\sum_{l=1}^{12} f(l; \pi\pi) = \sum_{l=1}^{12} f(l; K\pi) = \sum_{l=1}^{12} f(l; q\bar{q}) = 1.$$
(5.35)

 $f(l;\pi\pi)$ is determined from the signal Monte Carlo, and we assume $f(l;K\pi) = f(l;\pi\pi)$ because the LR and r distributions for $B^0 \to \pi^+\pi^-$ and $B^0 \to K^+\pi^-$ are identical because of the similar event topologies. $f(l;q\bar{q})$ is estimated using the number of events in the sideband region of the real data.

We calculate $n_{\pi\pi}$, $n_{K\pi}$ and $n_{q\bar{q}}$ such that the S/N ratio for the higher LR regions $(l = 7 \sim 12)$ is consistent with the ratio of the signal to the background in the real data, which is derived from the ΔE distribution (Section 4.3.3, Table 4.7), i.e.

$$n_{\pi\pi} \equiv N(B^0 \to \pi^+ \pi^-) / \sum_{l=7}^{12} f(l; \pi\pi), \qquad (5.36)$$

$$n_{K\pi} \equiv N(B^0 \to K^+\pi^-) / \sum_{l=7}^{12} f(l; K\pi)$$
(5.37)

and

$$n_{q\bar{q}} \equiv N(q\bar{q}) / \sum_{l=7}^{12} f(l; q\bar{q}).$$
(5.38)

The obtained values of $g_{\pi\pi}^l$, $g_{K\pi}^l$ and $g_{\pi\pi}^l$ are listed in Table 5.4. By using obtained $g_{\pi\pi}^l$, $g_{K\pi}^l$ and $g_{\pi\pi}^l$, the yields of the $B^0 \to \pi^+\pi^-$ events, $B^0 \to K^+\pi^-$ and continuum background in 485 candidates in the lower \mathcal{LR} region are estimated to be 57 ± 8 , 22 ± 6 and 406 ± 17 , respectively, while the yields of the $B^0 \to \pi^+\pi^-$ events, $B^0 \to K^+\pi^-$ and continuum background in 275 candidates in the higher \mathcal{LR} region are 106^{+16}_{-15} , 41^{+10}_{-9} and 128^{+5}_{-6} , respectively.



Figure 5.3: The ΔE distribution in the lower \mathcal{LR} region. The open circles represent the data. The broken line, the dash-dotted line, the dotted line and the solid-line histogram show the $B^0 \to \pi^+\pi^-$, $B^0 \to K^+\pi^-$ background, continuum background and the other B meson decays, respectively. The solid-line curve represents the sum of the four components.

5.2.6 Result of Fit

We extract the central values of $\mathcal{A}_{\pi\pi}$ and $\mathcal{S}_{\pi\pi}$ from the Δt distribution for the 760 final candidates (391 B^0 tagged and 369 \overline{B}^0 -tagged candidates) that contain $163^{+24}_{-23} B^0 \to \pi^+\pi^-$ signal events. In the fit, $\mathcal{A}_{\pi\pi}$ and $\mathcal{S}_{\pi\pi}$ are the free parameters, and the other parameters are fixed. There are 86 fixed parameters, including 3 physics parameters (τ_{B^0} , Δm_d and $\mathcal{A}_{K\pi}$), 6 wrong tag fractions (Table 4.9), 22 Δt resolution function parameters ($\beta_{\Upsilon(4S)}$,

Lower $\mathcal{L}\mathcal{K}$ regions		Higher $\mathcal{L}\mathcal{K}$ regions	
l	c_1	l	c_1
1	-0.270 ± 0.017	7	-0.109 ± 0.031
2	$-0.375 \ {}^{+}_{-} \ {}^{0.034}_{0.033}$	8	$-0.266 \ {}^+ \ {}^{0.063}_{0.062}$
3	-0.419 ± 0.036	9	$-0.312 \ ^+ \ ^0.080 \ ^0.079$
4	$-0.478 \ ^+ \ ^0.035 \ ^0.034$	10	$-0.378 \ {}^{+}_{-} \ {}^{0.077}_{0.075}$
5	$-0.541 \ {}^+_{-} \ {}^{0.047}_{0.045}$	11	-0.46 ± 0.11
6	$-0.578 \stackrel{+}{_{-}} \stackrel{0.071}{_{-}} \stackrel{0.071}{_{-}}$	12	-0.24 ± 0.18

Table 5.3: The slope parameter of the ΔE distribution for continuum background depending on the \mathcal{LR} -r region.

Table 5.4: The fractions of expected $B^0 \to \pi^+\pi^-$, $B^0 \to K^+\pi^-$ and background events for 12 *LR-r* regions. $g_{K\pi}^l = (0.382^{+0.112}_{-0.109}) \times g_{\pi\pi}^l$.

Lower \mathcal{LR} regions		Higher \mathcal{LR} regions			
l	$g^l_{\pi\pi}$	$g^l_{qar q}$	l	$g^l_{\pi\pi}$	$g^l_{qar{q}}$
1	0.087 ± 0.034	0.880 ± 0.040	7	0.296 ± 0.077	0.591 ± 0.028
2	0.127 ± 0.049	0.824 ± 0.040	8	0.385 ± 0.094	0.468 ± 0.026
3	0.124 ± 0.036	0.829 ± 0.041	9	0.407 ± 0.134	0.438 ± 0.027
4	0.129 ± 0.050	0.822 ± 0.040	10	0.442 ± 0.110	0.389 ± 0.024
5	0.170 ± 0.060	0.765 ± 0.040	11	0.522 ± 0.081	0.279 ± 0.022
6	0.390 ± 0.098	0.461 ± 0.032	12	0.670 ± 0.129	0.074 ± 0.009

 m_B , Table 4.11, 4.12 and 5.1), 11 parameters for the Δt shape of the continuum background (Table 5.2) and 44 parameters for signal and background probabilities (Equation 4.7, 4.8 and 4.9, α , Table 5.3 and 5.4). There are the 9 event-by-event variables other than Δt and q, including 5 kinematical variables ($M_{\rm bc}$, ΔE , $E_{\rm beam}^*$, $|\mathbf{p}_B^*|$, and $\cos \theta_B^*$) and 4 vertex measurement qualities ($\sigma_{z_{CP}}$, $\sigma_{z_{\rm tag}}$, $\xi_{z_{CP}}$ and $\xi_{z_{\rm tag}}$). The fit yields

$$\mathcal{A}_{\pi\pi} = +0.77 + 0.20 \, (\text{stat}^*) \tag{5.39}$$

and

$$S_{\pi\pi} = -1.23 + 0.24_{-0.15} \,(\text{stat}^*) \tag{5.40}$$

where the statistical errors³ are calculated by scanning the contour of Equation 5.3 using MINOS [115]. The values obtained reside outside of physical boundary. The correlation coefficient between $\mathcal{A}_{\pi\pi}$ and $\mathcal{S}_{\pi\pi}$ is calculated as 0.024. As described later, the statistical errors in the above equations are smaller than that expected from the Monte Carlo simulation. We discuss the statistical power of our measurement at the Section 5.3. Figure 5.4(a) shows the Δt distributions for the all candidates and the fit results in both flavor assignments. The fitted curves show the good agreement with the data. Figure 5.4(b) (Figure 5.4(c)) shows the Δt distributions for the candidates in the higher (lower) \mathcal{LR} region and the fit results obtained by the fit to the entire candidate. The fitted curves represent the Δt distributions in both \mathcal{LR} regions. Δt distributions after subtracting the background for both flavor assignments are shown in Figure 5.5(a), while the same plots for the higher (lower) \mathcal{LR} region are also shown in Figure 5.5(b) (Figure 5.5(c)). The difference of the signal yield between two flavor assignments is significantly large. The asymmetries of the $B^0 \to \pi^+\pi^-$ yield between q = +1 and q = -1 candidates are also shown in Figure 5.4.

5.3 Statistical Uncertainties

As described in Section 5.2.6, we get the statistical error for $\mathcal{A}_{\pi\pi}$ ($\mathcal{S}_{\pi\pi}$) of $^{+0.20}_{-0.23}$ ($^{+0.24}_{-0.15}$) from the contour of the likelihood functions. The expected values for $\mathcal{A}_{\pi\pi}$ ($\mathcal{S}_{\pi\pi}$) obtained by the ensemble test using the Monte Carlo pseudo-experiments is $^{+0.24}_{-0.26}$ ($^{+0.36}_{-0.32}$), where the Monte Carlo pseudo-experiments are carried out with input values of $\mathcal{A}_{\pi\pi} = +0.53$ and $\mathcal{S}_{\pi\pi} = -0.85$, which correspond to the values at the physical boundary closest to our central values. Here, the statistical errors of $\mathcal{A}_{\pi\pi}$ and $\mathcal{S}_{\pi\pi}$ estimated from the contour of the likelihood functions are significantly smaller than the expectations from the Monte Carlo. Figure 5.6 shows comparison of the statistical

³The statistical errors for $\mathcal{A}_{\pi\pi}$ and $\mathcal{S}_{\pi\pi}$ calculated by scanning the contour of the likelihood functions are expressed as (stat *) hereafter.


Figure 5.4: Δt distributions of the q = +1 candidates (top) and q = -1 candidates (below), respectively. Filled circles represent the data and solid line curves show the fitted PDFs. Hatched area show the $B^0 \to \pi^+\pi^-$ components of the PDFs while the dashed line curves represent the background components of the PDFs.



Figure 5.5: Δt distributions of the q = +1 candidates (open circles) and q = -1 candidates (filled triangles) after subtracting the background (top) and the asymmetry of the $B^0 \to \pi^+\pi^-$ yield between q = +1 and q = -1 candidates (bottom). In above plots, the solid (dashed) line curves show the results of the $B^0 \to \pi^+\pi^-$ components of PDFs for q = +1 (q = -1) candidates obtained by the fit. In below plots, the solid line curves show the resultant *CP* asymmetry, while the dashed (dotted) line curves are the contribution from the cosine (sine) term.

errors from fit to data to the expected values in Monte Carlo pseudo-experiments. These characteristics are reproduced in the Monte Carlo pseudo-experiments, and the fraction of the events of which the error of $\mathcal{A}_{\pi\pi}$ ($\mathcal{S}_{\pi\pi}$) is smaller than the result in the real data is calculated as 1.1% (11.9%) in Monte Carlo pseudo-experiments.

Figure 5.7 shows the logarithmic likelihood values as functions of $\mathcal{A}_{\pi\pi}$ and $\mathcal{S}_{\pi\pi}$ normalized by the maximum values of the likelihood. The obtained logarithmic likelihood curves are deviated from parabola functions. It indicates the statistical error estimated from the contour of the likelihood, which is based on the Gaussian approximation of the likelihood curve, is not appropriate. Therefore we estimate the statistical errors from the RMS. of the distributions of $\mathcal{A}_{\pi\pi}$ and $\mathcal{S}_{\pi\pi}$ in the Monte Carlo pseudo-experiments. We obtain the statistical errors for $\mathcal{A}_{\pi\pi}$ and $\mathcal{S}_{\pi\pi}$ as ± 0.27 and ± 0.41 , respectively. Thus,

$$\mathcal{A}_{\pi\pi} = +0.77 \pm 0.27 (\text{stat}) \tag{5.41}$$

and

$$S_{\pi\pi} = -1.23 \pm 0.41 (\text{stat}).$$
 (5.42)

We describe an investigation of the source of the small errors in Section 5.5.



Figure 5.6: The distributions of statistical errors in Monte Carlo pseudo-experiments with input values of $A_{\pi\pi} = +0.53$ and $S_{\pi\pi} = -0.85$. The arrows indicate the results of the fit to real data sample.



Figure 5.7: The value of $-2\ln(L/L_{\text{max}})$ as a function of $\mathcal{A}_{\pi\pi}$ (a) and $\mathcal{S}_{\pi\pi}$ (b), where L_{max} is the maximum value of the likelihood function. The dotted lines are the parabolic lines.

5.4 Systematic Uncertainties

We estimate the systematic uncertainties by varying the parameters in the event reconstruction and the CP fitting procedures. We consider the following sources contribute to the systematic error, as summarized in Table 5.5.

5.4. SYSTEMATIC UNCERTAINTIES

1. Wrong tag fraction

The systematic uncertainty in the wrong tag fraction is estimated varying the wrong tag fraction for each r region individually, and summing them up in quadrature. The systematic error due to the possible difference between the q = +1 tag and the q = -1 tag is also estimated using the wrong tag fractions measured separately for $q = \pm 1$ listed in Table 5.6.

2. Physics parameters

The B^0 meson lifetime and the $B^0-\overline{B}^0$ mixing parameter are fixed to the world averages (Equation 5.7 and Equation 5.8). The systematic uncertainty is estimated by varying the values within their errors. In this analysis, the direct CP asymmetry of $B^0 \to K^+\pi^-$ decays, $\mathcal{A}_{K\pi}$, is also fixed to be zero. Since the value of

$$\mathcal{A}_{K\pi} = -0.07 \pm 0.06 \tag{5.43}$$

is obtained from the self-tagged $B^0 \to K^+\pi^-$ sample with the 78 fb $^{-1}$ data [116], the systematic error is estimated by varying $\mathcal{A}_{K\pi}$ from -0.13 to +0.06.

3. Resolution of Δt measurement.

The uncertainty in the signal Δt resolution is estimated by varying each parameter individually by 1σ for the parameter determined by the real data and 2σ for those determined by Monte Carlo, and summing them up in quadrature.

4. Background Δt shape

The systematic error in the Δt shape of the continuum background is estimated by varying the parameters in $P^i_{q\bar{q}}(\Delta t;q)$ (Equation 5.23) by their errors.

5. Background fraction

The event-by-event fractions of the signal, the $B^0 \to K^+\pi^-$ background, and the continuum background are determined by the $(\Delta E, M_{\rm bc})$ shape, the yield of each component in the higher \mathcal{LR} region and the event fractions of 12 \mathcal{LR} -r regions.

The systematic errors on the $(\Delta E, M_{\rm bc})$ shape and the yield of each component in the higher \mathcal{LR} region is estimated by varying each parameter individually by 1σ for the parameter determined by the real data and 2σ for those determined by Monte Carlo, then summing them up in quadrature.

For the signal and the $B^0 \to K^+\pi^-$ background, the event fractions of 12 \mathcal{LR} -r regions are determined by the Monte Carlo. The difference between the Monte Carlo and the real data is investigated using $B \to D^{(*)}\pi$ decays. The systematic errors on the event fractions of 12 \mathcal{LR} -r regions is estimated by varying the fractions by this difference and adding the statistical errors of the $B \to D^{(*)}\pi$ real data in quadrature.

For the $B^0 \to K^+\pi^-$ background, we estimate the number of background events in the signal region as $32 \pm 2 \ (15 \pm 2)$ in the higher (lower) \mathcal{LR} region using the $B^0 \to K^+\pi^-$ sample and the kaon mis-identification probability that is estimated from the inclusive D^* -tagged $D^0 \to K^-\pi^+$ and $\phi \to K^+K^-$ decays. This is consistent with the number of events from a fit to the ΔE distribution. The number of background events obtained from the kaon mis-identification probability changes the result for $\mathcal{A}_{\pi\pi}$ ($\mathcal{S}_{\pi\pi}$) by $^{+0.005}_{-0.0}$ ($^{+0.0}_{-0.03}$). This difference is included in the systematic error.

For the continuum background, the event fractions of 12 \mathcal{LR} -r regions are determined by the data in the mass sideband. The systematic error is estimated by varying the fractions by the statistical errors. The PDF for the continuum assumes the equal fractions of the events in both q = +1 and q = -1 samples, i.e. $\delta_{q\bar{q}} = 0$ in Equation 5.23. The systematic error due to this assumption is estimated by varying $\delta_{q\bar{q}}$ from 0.5 by ± 0.02 , based on the measured background asymmetry of 1.26 ± 0.63 obtained by counting the sideband data, described in Section 5.2.4.

6. Fit bias

We perform the CP fit to the signal events in the full Monte Carlo simulation based on the GEANT. The differences among the input values and the fitted results of $\mathcal{A}_{\pi\pi}$ and $\mathcal{S}_{\pi\pi}$ are included in the systematic error. The difference among the input values and the mean fitted values in the Monte Carlo pseudo-experiments are also included in the systematic errors.

7. Vertex reconstruction

We estimate the systematic uncertainty on the vertex reconstruction by varying the vertex quality requirements in the B_{CP} and the B_{tag} vertex reconstructions and the track requirements in the B_{tag} reconstruction. The difference of the fit results of $\mathcal{A}_{\pi\pi}$ and $\mathcal{S}_{\pi\pi}$ from the default values are included in the systematic error. We change the cut value of the vertex quality, ξ_{zCP}^i and ξ_{zCP}^i , to 50 and 200, respectively, where the default values are set to be 100. In the B_{tag} vertex reconstruction, the track requirement on the error of position in z is varied by 0.1 mm, where the default is < 0.5 mm, and the requirement on the distance from the reconstructed decay vertex of B_{CP} is varied by 0.1 mm, where the default is < 0.5 mm. We also estimate the systematic uncertainty on the IP constraint in the vertex reconstruction by varying the expected B meson flight length by +20 μ m and -10 μ m. We also repeat the analysis by introducing charge-dependent shifts in the z direction for tracks artificially and include the resulting change in the systematic error. Here the amount of the shift, which is $\pm 3 \ \mu m$ ($\mp 3 \ \mu m$) for the positive (negative) tracks, is determined from studies with cosmic rays and with the two-photon $e^+e^- \rightarrow \pi^+\pi^-\pi^+\pi^-$ process.

The total systematic errors of $\mathcal{A}_{\pi\pi}$ and $\mathcal{S}_{\pi\pi}$ are ± 0.08 and $^{+0.08}_{-0.07}$, respectively, by adding all these contributions in quadrature. The systematic error for $\mathcal{S}_{\pi\pi}$ is mainly from uncertainties in the background fraction and a possible fit bias near the physical boundary. For $\mathcal{A}_{\pi\pi}$, the background fraction and the vertex reconstruction are the two leading components. The systematic errors for both $\mathcal{A}_{\pi\pi}$ and $\mathcal{S}_{\pi\pi}$ are smaller than the statistical errors.

Table 5.5: Summary table of the systematic uncertainties for $\mathcal{A}_{\pi\pi}$ and $\mathcal{S}_{\pi\pi}$				
	\mathcal{A}	$\pi\pi$	S	$\pi\pi$
Source	positive error	negative error	positive error	negative error
Background fraction	+0.058	-0.048	+0.044	-0.055
Vertex reconstruction	+0.044	-0.054	+0.037	-0.012
Fit bias	+0.016	-0.021	+0.052	-0.020
Wrong tag fraction	+0.026	-0.021	+0.015	-0.016
Physics parameters	+0.021	-0.014	+0.022	-0.022
Resolution function	+0.019	-0.020	+0.010	-0.013
Background shape	+0.003	-0.015	+0.007	-0.002
Total	+0.084	-0.083	+0.083	-0.067

Table 5.6: The wrong tag fractions for q = +1 and q = -1. The errors are the statistical errors.

l	r	w_{\cdot}		
		q = +1	q = -1	Difference
1	0.000 - 0.250	0.4625 ± 0.0072	0.4535 ± 0.0073	$+0.009 \pm 0.010$
2	0.250 - 0.500	0.339 ± 0.011	0.333 ± 0.011	$+0.006 \pm 0.016$
3	0.500 - 0.625	0.211 ± 0.012	0.246 ± 0.012	-0.035 ± 0.017
4	0.625 - 0.750	0.148 ± 0.010	0.173 ± 0.011	-0.025 ± 0.015
5	0.750 - 0.875	0.101 ± 0.011	0.122 ± 0.011	-0.021 ± 0.016
6	0.875 - 1.000	0.0200 ± 0.0064	0.0196 ± 0.0062	$+0.0004 \pm 0.0089$

5.5 Validation Checks

We perform a number of checks in order to confirm the validity of our measurement:

1. Ensemble test

We check the validity of our fitting procedure using a large ensemble of Monte Carlo pseudo-experiments. We generate Monte Carlo pseudo-experiments containing the same number of events as the candidates in the real data. The events in Monte Carlo pseudo-experiment are based on the PDFs, the fractions of $B^0 \to \pi^+\pi^-$, $B^0 \to K^+\pi^-$ and continuum components and the wrong tag fractions obtained from the real data, which are used in the unbinned-maximum likelihood fit. We check linearities of $\mathcal{A}_{\pi\pi}$ and $\mathcal{S}_{\pi\pi}$ obtained by the fit with respect to the true values. Figure 5.8 shows the fit results of $\mathcal{A}_{\pi\pi}$ and $\mathcal{S}_{\pi\pi}$ as functions of the input

5.5. VALIDATION CHECKS

values, and fit results using a linear function. The results indicate good linearity and an absence of any bias. We generate the Monte Carlo pseudo-experiments with input values of $\mathcal{A}_{\pi\pi} = 0.53$ and $\mathcal{S}_{\pi\pi} = -0.85$, which correspond to the point in the physical boundary closest to our measurement, and the comparison with our measurement as shown in Figure 5.9. Figure 5.9(a) (5.9(b)), which is the distribution of $\mathcal{A}_{\pi\pi}$ ($\mathcal{S}_{\pi\pi}$), indicates the finite probability to obtain the fit results outside the physical boundary as our measurement. The probabilities to obtain the fit result outside the physical boundary and to exceed the *CP* violation are calculated as 14.8% and 59.8%, respectively, as shown in Table 5.7. Figure 5.9(c) is the distribution of $-2\ln(L_{\max})$, where L_{\max} is the maximum value of the likelihood function. $-2\ln(L_{\max})$ of our measurement is consistent with the Monte Carlo pseudo-experiments. The correlation coefficient of our measurement is also consistent with the expectation as shown in Figure 5.9(d).

2. Test using the non-CP sample

We extract " $\mathcal{A}_{\pi\pi}$ " and " $\mathcal{S}_{\pi\pi}$ " values from the flavor-specific decay final states in the real data to verify the procedures for the flavor tagging, the Δt reconstruction, the resolution function and the fit procedure. Since the flavor-specific decay modes are not expected to give the CP violation in this analysis, " $\mathcal{A}_{\pi\pi}$ " and " $\mathcal{S}_{\pi\pi}$ " values should be zero. The obtained " $\mathcal{A}_{\pi\pi}$ " and " $\mathcal{S}_{\pi\pi}$ " using $B \to D^{(*)-}\pi^+$ and $B \to D^{*-}\rho^+$ decays are consistent with zero, as shown in Table 5.8 and Figure 5.10(a). We also apply the same CP fitting procedure to $B^0 \to K^+\pi^-$ decays. For $B^0 \to K^+\pi^-$ decays, " $\mathcal{S}_{\pi\pi}$ " should be zero, although " $\mathcal{A}_{\pi\pi}$ " may be the non-zero value. The fit result of " $\mathcal{S}_{\pi\pi}$ " to $B^0 \to K^+\pi^-$ decays is measured to be consistent with zero, as shown in Table 5.8 and Figure 5.10(b). We can make the similar test using the $B^0 \to \pi^+\pi^-$ and the $B^0 \to K^+\pi^-$ candidates by assuming all candidates have a flavor of q = +1 and the CP fit with fixing $\mathcal{A}_{\pi\pi}$ to be zero (*flavor blind fit*). The obtained " $\mathcal{S}_{\pi\pi}$ "s are consistent with zero, as shown in Table 5.8. These results indicate the absence of significant bias in out procedure to determine $\mathcal{A}_{\pi\pi}$ and $\mathcal{S}_{\pi\pi}$. We also examine the event yield, and the Δt distribution for the events for each flavor assignment, q = +1 and q = -1, in the sideband region and find no significant asymmetry as shown in Figure 5.10(c).

3. B^0 lifetime and Δm_d measurements

To confirm the decay vertices of B mesons are successfully reconstructed, we perform the lifetime measurement using the same data sample. The lifetime of B^0 mesons is measured to be $1.42^{+0.14}_{-0.12}$ ps using 760 $B^0 \rightarrow \pi^+\pi^-$ candidates. The lifetime is also measured to be 1.46 ± 0.08 ps using 1371 $B^0 \rightarrow K^+\pi^-$ candidates. Both are consistent with the world average (Equation 5.7) and this indicates that our vertex reconstruction in the two-body B-meson decays is correct and that the Δt resolution is understood in good shape.

We also perform the Δm_d measurement using $B^0 \to K^+\pi^-$ decays, which are the flavor-specific two-body decays of B mesons. The measured value is $\Delta m_d = 0.55^{+0.05}_{-0.07} \hbar \text{ ps}^{-1}$, and consistent with the world average (Equation 5.8). This assures that our flavor-tagging method and the wrong-tag fraction estimation are correct.

4. Fit with the constraint to the physical boundary.

Since our nominal fit result is outside of the physical region, $\mathcal{A}_{\pi\pi}^2 + \mathcal{S}_{\pi\pi}^2 \leq 1$, we also perform the fit with a constraint that restricts the fit results into the physical region. The obtained results with the constraint are $\mathcal{A}_{\pi\pi} = +0.57$ and $\mathcal{S}_{\pi\pi} = -0.82$ on the boundary, which are consistent with the point on the boundary closest to the nominal values ($\mathcal{A}_{\pi\pi} = +0.53$, $\mathcal{S}_{\pi\pi} = -0.85$). Note that there is the disadvantage of such a fitting method because the errors returned from the fit are not Gaussian and are difficult to interpret when the fit result is close to the physical boundary.

5. Comparison with Time-independent analysis

We check the measurement of $\mathcal{A}_{\pi\pi}$ using the time-independent fits to the ΔE distributions for B^0 and \overline{B}^0 tags. Figure 5.11(a) and 5.11(b) show the ΔE distributions of the q = +1 and q = -1 candidates in the higher \mathcal{LR} region, respectively. The $B^0 \to \pi^+\pi^-$ signal yields for q = +1 and q = -1 candidates are 61.2 ± 11.4 and 43.7 ± 10.1 , respectively. We determine the yield of $B^0 \to \pi^+\pi^-$ signals by fitting ΔE distribution in 24 regions corresponding to 12 \mathcal{LR} -r regions for B^0 and \overline{B}^0 tags, as listed in Table 5.9. For each r-bin, $\mathcal{A}^l_{\pi\pi}$ is given by

$$\mathcal{A}_{\pi\pi}^{l} \equiv \frac{1}{(1-2\chi_{d})(1-2w_{\rm tag}^{l})} \cdot \frac{[N_{\pi\pi}^{l}(q=1) - N_{\pi\pi}^{l}(q=-1)]}{[N_{\pi\pi}^{l}(q=1) + N_{\pi\pi}^{l}(q=-1)]},$$
(5.44)

where χ_d is a time integrated mixing probability of 0.181 ± 0.004 [42], and $N_{\pi\pi}^l(q = \pm 1)$ is $B^0 \to \pi^+\pi^-$ yield in *r*-region with flavor assignment of $q = \pm 1$. The weighted mean with the statistical errors of $\mathcal{A}_{\pi\pi}^l(l = 1 \sim 6)$ gives $\mathcal{A}_{\pi\pi}$ corrected for w_{tag}^l and χ_d . We obtain $\mathcal{A}_{\pi\pi} = 0.55 \pm 0.37$ (stat) from the time-independent analysis, which is consistent with the result from the fit of the Δt distribution.

6. ΔE shapes for $B^0 \to \pi^+\pi^-$ and $B^0 \to K^+\pi^-$

In calculation of the signal probabilities, which is described in Section 5.2.5, the ΔE distributions for $B^0 \to \pi^+\pi^-$ and $B^0 \to K^+\pi^-$ are modeled by a single Gaussian function. Here we use the sum of two Gaussians with the same mean values to model ΔE distributions for $B^0 \to \pi^+\pi^-$ and $B^0 \to K^+\pi^-$. The values of width of Gaussians are obtained using the same procedure in Section 4.3.3 while the fraction of second Gaussian is obtained using a GEANT based Monte Carlo simulation as 0.17 for $B^0 \to \pi^+\pi^-$ or 0.13 for $B^0 \to K^+\pi^-$. Figure 5.13(a) shows the fit of ΔE distribution in the higher \mathcal{LR} region with a double Gaussian parameterization. The CP fit with this double Gaussian parameterization gives $\mathcal{A}_{\pi\pi} = 0.75^{+0.20}_{-0.22}(\text{stat}^*)$ and $\mathcal{S}_{\pi\pi} = -1.21^{+0.23}_{-0.14}(\text{stat}^*)$, which are consistent with the default fit results. We also apply a CP fit with modeling the ΔE distribution for $B^0 \to \pi^+\pi^-$ and $B^0 \to K^+\pi^-$ by a single Gaussian at the sum of two Gaussians, respectively, as shown in Figure 5.13(b). The results are $\mathcal{A}_{\pi\pi} = 0.77^{+0.20}_{-0.23}(\text{stat}^*)$ and $\mathcal{S}_{\pi\pi} = -1.22^{+0.23}_{-0.15}(\text{stat}^*)$, and also consistent with the default fit results.

7. Δt shape of continuum backgrounds.

For continuum backgrounds, we use the same Δt shape for B^0 and $\overline{B}{}^0$ tagged events in the probability density function, as described in Section 5.2.4. Although the flavor asymmetry for the continuum background is small, as shown in Figure 5.10(c), we check the effect to the *CP* fitting. We use the probability density function for continuum background defined as Equation 5.23 as the default. Here we use different $f_{\delta}^{q\bar{q}}$ and $\tau_{q\bar{q}}$ for each flavor assignment, and obtain $\mathcal{A}_{\pi\pi} = 0.77^{+0.20}_{-0.23}(\text{stat}^*)$ and $\mathcal{S}_{\pi\pi} = -1.23^{+0.24}_{-0.15}(\text{stat}^*)$ with the background-yield asymmetry of $\delta_{q\bar{q}} = 0$. These fit results are same as the default fit results. The fit results with $\delta_{bkg} = +0.02$ are $\mathcal{A}_{\pi\pi} = 0.77^{+0.21}_{-0.23}(\text{stat}^*)$ and $\mathcal{S}_{\pi\pi} = -1.23^{+0.24}_{-0.15}(\text{stat}^*)$, which are also consistent with the default.

8. Fit of subsamples with tighter event selection

We apply more stringent selection criteria for $B^0 \to \pi^+\pi^-$ candidates and perform the *CP* fit. We do not observe any systematic tendency when we vary the selection on \mathcal{LR} , ΔE , r, vertex quality and kaon probability as shown in Table 5.10. Restricting the ΔE range or applying much more restrictive kaon probability requirements that reduce the $B^0 \to K^+\pi^-$ background do not significantly change the result.

9. The difference between the error obtained from the fit to the data and that expected from the Monte Carlo pseudo-experiments

We investigate the reason why the error of $S_{\pi\pi}$ estimated from the likelihood function is smaller than the expectation from Monte Carlo pseudo-experiments. In Monte Carlo pseudo-experiments, we find that the logarithmic likelihood curve may deviates from a parabola function when the number of events is not large and the true values of $S_{\pi\pi}$ and $A_{\pi\pi}$ are located close to a physical boundary. In such a case, a small number of candidates can have a large influence on both the size of the error⁴ and the shape of the logarithmic likelihood curve. The likelihood function for some candidates may become negative when the fit parameter is beyond the physical boundary.

The observed features of the errors arise when there is an candidate that restricts the fit parameters in or close to the physical region, while the fit to all the other candidates gives a maximum likelihood that is located outside the physical region and is not allowed by the aforementioned restrictive candidate. For example, in this fit the removal of such a restrictive candidate results in a $S_{\pi\pi}$ value that is more negative than $S_{\pi\pi} = -1.23$ (further from the physical boundary). In this case, the logarithmic likelihood curve is deformed by an inclusion of the restrictive candidate, even if the curve before the inclusion is well described by a parabola. The sizes of the errors also become small.

We investigate this type of single-event sensitivity and its relation to the size of the errors with Monte Carlo pseudo-experiments. For each experiment, we repeat the fit by removing each candidate in turn. The candidate that creates the largest difference in $S_{\pi\pi}$ is tagged as the restrictive candidate and the change produced by the removal of the restrictive candidate, $\Delta S_{\pi\pi}$, is recorded. When we choose the point of $(\mathcal{A}_{\pi\pi}, \mathcal{S}_{\pi\pi}) = (+0.53, -0.85)$, which is closest to our central values, as the input values to the Monte Carlo pseudo-experiments, we obtain the average values of $\Delta S_{\pi\pi}$ as a function of the positive error of $\mathcal{S}_{\pi\pi}$ shown in Figure 5.14. The correlation between the size of the error and the single-event sensitivity is evident.

⁴In this context, "error" means the error estimated from the likelihood function.

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In our data, we have one candidate that has a large effect on the sizes of errors. The removal of this candidate from the fit gives $S_{\pi\pi} = -1.91^{+0.36}_{-0.33}(\text{stat}^*)$ and $\mathcal{A}_{\pi\pi} = 0.64^{+0.19}_{-0.20}(\text{stat}^*)$, where $S_{\pi\pi}$ is shifted to the negative direction ($\Delta S_{\pi\pi} = -0.67$) and the error increases with respect to the nominal fit results (Equation 5.40). This candidate has qr = -0.92 which is close to unambiguous *B* flavor assignment and corresponds to a very small wrong-tag probability. In addition, this candidate has $\Delta E = -0.01$ GeV, and $\mathcal{LR} = 0.98$, which corresponds to small $B^0 \to K^+\pi^-$ and continuum background probabilities. Thus, the dilution factor for this candidate is close to the unity. Moreover, for this candidate, $\Delta t = -3.8$ ps and, it gives $\sin(\Delta m_d \Delta t) \simeq -1$. According to Equation 5.6 using the above conditions, this candidate has a negative likelihood value at negative $S_{\pi\pi}$ values beyond ~ -1.5 , where it truncates the logarithmic likelihood ratio curve as shown in Figure 5.15. As a result, the negative error for the entire event sample is restricted by this single event. This phenomenon is reproduced in the Monte Carlo pseudo-experiments. This type of single-candidate sensitivity occurs in a few percent when the fit result is outside the physical region.

As shown in Figure 5.14, the observed single-event sensitivity $\Delta S_{\pi\pi} = -0.67$ is consistent with the expectation from the Monte Carlo pseudo-experiments if the positive error of $S_{\pi\pi}$ is ~ +0.24, which is the case for our data. A similar study for input values of $S_{\pi\pi}$ and $A_{\pi\pi}$ that are well within the physically allowed region indicates that this behavior occurs much less often.



Figure 5.8: Mean values of the fit result vs. input values of Monte Carlo pseudo-experiments. The solid lines are linear fit results.

	~ -		
1	The fractions outside physical	The fractions exceed the CP vi-	
Input $\sqrt{A_{\pi\pi}} + \mathcal{S}_{\pi\pi}$	boundary we observe $(\%)$	olation we observe $(\%)$	
0.00	1.8	0.07	
0.00 - 0.20	2.5	0.12	
0.20 - 0.40	4.7	0.3	
0.40 - 0.60	11.2	1.0	
0.60 - 0.80	24.3	3.1	
0.80 - 1.00	45.9	9.2	
1.00	59.3	14.8	

Table 5.7: The fractions of Monte Carlo pseudo-experiments outside physical boundary and exceed the CP violation we observe for various input values.



Figure 5.9: Results of the Ensemble test with Monte Carlo pseudo-experiments with input values of the $A_{\pi\pi} = 0.53$ and $S_{\pi\pi} = -0.85$. The solid line arrow and the dashed line arrow indicate the input value of the Monte Carlo pseudo-experiments and the result of the fit of the real data, respectively.



Figure 5.10: The flavor asymmetry in the Δt distributions for control samples.

Decay mode	# of events	$``\mathcal{A}_{\pi\pi}"$	" $\mathcal{S}_{\pi\pi}$ "
$B \rightarrow D^- \pi^+$	5,229	$+0.009 \pm 0.035$	$+0.083 \pm 0.052$
$B \to D^{*-} \pi^+$	5,016	-0.012 ± 0.036	$+0.007 \pm 0.055$
$B \to D^{*-} \rho^+$	5,076	-0.051 ± 0.042	$+0.038 \pm 0.064$
(combined)	$15,\!321$	-0.015 ± 0.022	$+0.045 \pm 0.033$
$B^0 \to K^+ \pi^-$	1,371	-0.03 ± 0.11	$+0.08\pm0.16$
$B^0 \to K^+ \pi^- \ (\mathcal{LR} > 0.825)$	585	-0.01 ± 0.12	$+0.10\pm0.18$
$B^0 \to K^+ \pi^-$ (flavor blind)	1,371		$+0.03\pm0.16$
$B^0 \to \pi^+ \pi^-$ (flavor blind)	760		$-0.25 \ ^+ \ ^{0.35} _{0.34}$
$B^0 \to \pi^+\pi^-$ (flavor blind, $\mathcal{LR} > 0.825$)	275		$-0.10 \ ^+ \ ^0.38 _{- \ 0.37}$

Table 5.8: The CP fit results for non-CP samples. The errors are statistical error.



Figure 5.11: The ΔE distributions of the q = +1 candidates (a) and q = -1 candidates (b) in the higher \mathcal{LR} and $M_{\rm bc}$ signal region. The open circles are represents the data and the solid-line curves are represents the fitted results. The open circles are represents the data and the solid-line curves are represents the fitted results. The broken line, the dash-dotted line, the dotted line and the solid-line show the $B^0 \to \pi^+\pi^-$, $B^0 \to K^+\pi^-$ background, continuum background and the other B meson decays, respectively.

1 110	The chois by summation of the the obtained by adding each choir in quadratatic.						
l	r	$B^0 { m t}$	agging	$\overline{B}{}^0$ t	agging	B^0 -tag	$+ \overline{B}^0$ -tag
		$B^0 \to \pi^+\pi^-$	$B^0 \to K^+ \pi^-$	$B^0 \to \pi^+\pi^-$	$B^0 \to K^+ \pi^-$	$B^0 \to \pi^+\pi^-$	$B^0 \to K^+ \pi^-$
1	0.000 - 0.250	$25.3 \ {}^{+}_{-} \ {}^{8.0}_{7.3}$	$11.9 \ {}^{+}_{-} \ {}^{7.7}_{6.9}$	$22.0 \stackrel{+}{_{-}} \stackrel{7.5}{_{-}} \stackrel{7.5}{_{-}}$	$4.2 {}^{+}_{-} {}^{6.4}_{0.0}$	$47.3 \ {}^{+}_{-} \ {}^{11.0}_{6.8}$	$16.1 {}^{+}_{-} {}^{10.0}_{6.9}$
2	0.250 - 0.500	$7.5 \ {}^{+}_{-} \ {}^{4.7}_{4.0}$	$0.5 \ ^+_{-} \ ^{3.9}_{0.0}$	$7.6 \ {}^{+}_{-} \ {}^{4.7}_{4.0}$	$7.5 \ {}^{+}_{-} \ {}^{4.6}_{3.9}$	$15.1 {}^{+}_{-} {}^{6.6}_{5.7}$	$8.0 \ {}^+_{-} \ {}^{6.0}_{3.9}$
3	0.500 - 0.625	$1.1 \ ^+_{-} \ ^{3.0}_{0.0}$	$0.0 \ ^+ \ ^2.4 \ ^0.0$	$3.6 {}^{+}_{-} {}^{3.2}_{2.5}$	$0.0 \ ^{+}_{-} \ ^{2.1}_{0.0}$	$4.7 {}^{+}_{-} {}^{4.4}_{2.5}$	$0.0 \ ^{+}_{-} \ ^{3.2}_{0.0}$
4	0.625 - 0.750	$10.2 \ {}^{+}_{-} \ {}^{4.5}_{3.8}$	$0.9 \ ^+_{-} \ ^{3.3}_{0.0}$	$4.4^{+3.5}_{-2.8}$	$4.4 {}^{+}_{-} {}^{3.6}_{2.9}$	$14.6 \ {}^{+}_{-} \ {}^{5.7}_{4.7}$	$5.3 {}^{+}_{-} {}^{4.9}_{2.9}$
5	0.750 - 0.875	$4.7 \ {}^{+}_{-} \ {}^{3.2}_{2.6}$	$3.1 {}^{+}_{-} {}^{3.5}_{2.8}$	$0.0 \ ^+_{-} \ ^{1.6}_{0.0}$	$8.9 {}^{+}_{-} {}^{3.9}_{3.2}$	$4.7 {}^{+}_{-} {}^{3.6}_{2.6}$	$12.0 \ {}^{+}_{-} \ {}^{5.6}_{4.5}$
6	0.875 - 1.000	$11.6 \ {}^+ \ {}^{4.6}_{3.9}$	$11.3 \ {}^{+}_{-} \ {}^{4.7}_{3.9}$	$6.1 {}^+_{-2.6} {}^{3.2}_{-2.6}$	$3.6 {}^{+}_{-} {}^{3.0}_{2.2}$	$17.7 {}^{+}_{-} {}^{5.6}_{4.7}$	$14.9 \ {}^{+}_{-} \ {}^{5.6}_{4.5}$
	sum of $l = 1 \sim 6$	$60.4 \ ^+_{-10.3} \ ^+_{10.3}$	$27.7 {}^{+}_{-}{}^{8.3}_{8.4}$	$43.7 {}^{+}_{-}{}^{10.7}_{9.1}$	$28.6 \ ^+_{- \ 6.2} \ ^{10.2}$	$104.1 \ {}^{+}_{-} \ {}^{16.2}_{14.0}$	$56.3 {}^{+}_{-} {}^{13.2}_{10.4}$

Table 5.9: The number of $B^0 \to \pi^+\pi^-$ signal and $B^0 \to K^+\pi^-$ background in higher \mathcal{LR} region for each *r*-bin. The errors by summation of $l = 1 \sim 6$ are obtained by adding each error in quadrature.







(a) Δt distribution of $B^0 \to \pi^+\pi^$ candidates. The solid line shows the result of lifetime fit. The dashed line shows the $B^0 \to \pi^+\pi^-$ and $B^0 \to K^+\pi^-$ components of the fitted function. The dashed-dotted line shows the continuum background components of the fitted function.

(b) Δt distribution of $B^0 \to K^+\pi^$ candidates. The solid line shows the result of lifetime fit. The dashed line shows the $B^0 \to \pi^+\pi^-$ and $B^0 \to K^+\pi^-$ components of the fitted function. The dashed-dotted line shows the continuum background components of the fitted function.

(c) $B^{0}-\overline{B}^{0}$ measurement using the $B^{0} \rightarrow K^{+}\pi^{-}$ candidates. Data points shows Opposite Flavor(OF) - Same Flavor(SF) asymmetry as a function of Δt . The solid line shows the fit result.

Figure 5.12: The validation checks for the vertex reconstruction and the flavor tagging



(a) $B^0 \to \pi^+\pi^-$ and $B^0 \to K^+\pi^-$ components are modeled by the sum of double Gaussian.

(b) $B^0 \to \pi^+\pi^-$ and $B^0 \to K^+\pi^-$ components are modeled by the single Gaussian and the sum of double Gaussians, respectively.

Figure 5.13: Fit of ΔE distribution. The open circles and the solid-line curves represent the data and the fitted results, respectively. The broken line, the dash-dotted line, the dotted line and the solid-line histogram show the $B^0 \to \pi^+\pi^-$, $B^0 \to K^+\pi^-$ background, continuum background and the other B meson decays, respectively.

Category	criteria	# of candidate	$\mathcal{A}_{\pi\pi}$	$\mathcal{S}_{\pi\pi}$
Continuum suppression	$\mathcal{LR} > 0.825$	275	$+0.84 + 0.22 \\ - 0.25 (stat^*)$	$-1.19 \stackrel{+}{_{-}} \stackrel{0.27}{_{-}} (\text{stat}^*)$
	$\mathcal{LR} > 0.925$	131	$+0.69 \stackrel{+}{_{-}} \stackrel{0.26}{_{-}} (\mathrm{stat}^*)$	$-1.24 \stackrel{+}{_{-}} \stackrel{0.30}{_{0.19}} (\text{stat}^*)$
Δt region	$ \Delta t < 15 \text{ ps}$	755	$+0.77 \stackrel{+}{_{-}} \stackrel{0.20}{_{-}} (\text{stat}^*)$	$-1.25 \stackrel{+}{_{-}} \stackrel{0.24}{_{-}} (\text{stat}^*)$
	$ \Delta t < 5 \text{ ps}$	736	$+0.76 \stackrel{+}{_{-}} \stackrel{0.20}{_{-}} (\mathrm{stat}^*)$	$-1.27 \stackrel{+}{_{-}} \stackrel{0.26}{_{-}} (\text{stat}^*)$
Vertex reconstruction quality	$\xi < 50$	741	$+0.77^{+0.21}_{-0.23}(\text{stat}^*)$	$-1.22 \stackrel{+}{_{-}} \stackrel{0.25}{_{-}} (\text{stat}^*)$
	$\xi < 10$	564	$+0.91 \stackrel{+}{_{-}} \stackrel{0.20}{_{-}} (\text{stat}^*)$	$-1.09 \stackrel{+}{_{-}} \stackrel{0.32}{_{-}} (\text{stat}^*)$
ΔE signal region	$ \Delta E < 1\sigma$	298	$+0.82 \ ^{+0.21}_{-0.25}(\text{stat}^*)$	$-1.18 \stackrel{+ 0.29}{_{- 0.19}}(\text{stat}^*)$
	$ \Delta E < 2\sigma$	540	$+0.81 \stackrel{+}{_{-}} \stackrel{0.20}{_{-}} (\text{stat}^*)$	$-1.21 \stackrel{+}{_{-}} \stackrel{0.25}{_{0.16}} (\text{stat}^*)$
Flavor tagging quality	r > 0.75	97	$+1.02 \ ^{+0.19}_{-0.25}(\text{stat}^*)$	$-1.24 \stackrel{+}{_{-}} \stackrel{0.27}{_{-}} (\text{stat}^*)$
	r > 0.875	44	$+0.91 \stackrel{+}{_{-}} \stackrel{0.24}{_{-}} (\text{stat}^*)$	$-1.18 \stackrel{+ 0.32}{_{- 0.19}}(\text{stat}^*)$
Kaon identification	$L^{hh}(K;\pi) < 0.20$	596	$+0.74 \stackrel{+}{_{-}} \stackrel{0.20}{_{-}} (\text{stat}^*)$	$-1.11 \stackrel{+}{_{-}} \stackrel{0.26}{_{-}} (\text{stat}^*)$
	$L^{hh}(K;\pi) < 0.15$	556	$+0.59 \stackrel{+}{_{-}} \stackrel{0.22}{_{-}} (\text{stat}^*)$	$-1.14 \stackrel{+}{_{-}} \stackrel{0.23}{_{0.14}} (\text{stat}^*)$

Table 5.10: CP fit of sub samples with tighter event selection



Figure 5.14: Single-event sensitivity vs. the positive error on $S_{\pi\pi}$. The dashed lines indicate the observed $\Delta S_{\pi\pi}$ and $S_{\pi\pi}$ positive error.



Figure 5.15: $-2\ln(L/L_{\text{max}})$ as a function of $S_{\pi\pi}$ for the candidates except a certain candidate (solid line) and that for the whole candidates (dashed line), where L_{max} is the maximum value of the likelihood function for the candidates except the candidate (see text).

Chapter 6

Discussions and Conclusion

6.1 Statistical Significance

In this section, we evaluate the statistical significance of our CP violation measurement [117]. First, we describe the method to obtain the confidence interval. We then derive the confidence intervals for $\mathcal{A}_{\pi\pi}$ and $\mathcal{S}_{\pi\pi}$ separately. We also derive the confidence region in the $\mathcal{A}_{\pi\pi}$ - $\mathcal{S}_{\pi\pi}$ space.

6.1.1 Confidence interval

The confidence interval (or in the multi-parameter case, confidence region) is constructed such that it includes the true value of the parameter with a probability greater than or equal to the specified value, which is called the coverage probability. Consider a probability density function $p(x|\mu)$ where x represents the outcome of the experiment and μ is the unknown true value of the parameter for which we want construct a confidence interval. We can find a set of values $x_1(\mu, \alpha)$ and $x_2(\mu, \alpha)$ for the pre-specified probability $(1 - \alpha)$ and every values of μ such that

$$P(x_1 < x < x_2; \mu) \equiv \int_{x_1}^{x_2} dx \ p(x|\mu) = 1 - \alpha.$$
(6.1)

This is illustrated in Figure 6.1(a). A horizontal line segment $[x_1(\mu, \alpha), x_2(\mu, \alpha)]$ is drawn for the representative value of μ . The union of such intervals for all values of μ , designated in the figure as $D(\alpha)$, is referred to as the *confidence belt*. Upon performing an experiment to measure x and obtaining a value x_{exp} , one draws a vertical line through x_{exp} , as shown in Figure 6.1(b). The confidence interval for μ is the set of all values of μ on the intersection of the vertical line and the confidence belt. Such confidence interval is said to have a *confidence level* (CL) equal to $(1 - \alpha)$.

Note that the construction of the confidence interval is equivalent to considering a test of the hypothesis that parameter's true value is μ . One then excluded all values of μ where hypothesis would be rejected at a *significance level* less than α . The remaining values constitute the confidence interval at a confidence level $(1-\alpha)$. For a certain value of μ , we can calculate the confidence level to reject the hypothesis that the true value of the parameter is equal to μ , $CL(\mu)$, from the confidence interval. $CL(\mu)$ is determined so that μ locates just outside the confidence interval with the confidence level of $CL(\mu)$.

Equation 6.1 has the ambiguity in the determination of x_1 and x_2 . In this analysis, the integral interval is determined with the method based on the *likelihood ratio ordering principle*, which is proposed by G. Feldman and R. Cousins [114]. A test statistic based on the likelihood ratio is defined as:

$$\lambda(x;\mu) = \frac{p(x|\mu)}{p(x|\mu_{\text{best}})}$$
(6.2)

where μ_{best} is the value of the parameter which, out of all allowed values, maximizes $p(x|\mu)$. The upper and lower boundaries in Equation 6.1 are determined by the condition:

$$\lambda(x_1;\mu) = \lambda(x_2;\mu). \tag{6.3}$$

Thus, when the experimental value is obtained as x_{exp} , the confidence level to reject the hypothesis that the true value of the parameter is equal to μ is calculated as:

$$\operatorname{CL}(\mu) = \int_{\lambda(x;\mu) \le \lambda(x_{\exp};\mu)} \mathrm{d}x \ p(x|\mu).$$
(6.4)

Equation 6.2 and Equation 6.4 can be extended for the multi-parameter case. The confidence region in the multi-parameter case is obtained from the contour of $CL(\mu)$.



(a) Construction of the confidence belt from the probability density function.

(b) Construction of the confidence interval from the confidence belt and the experimental value.

Figure 6.1: Conceptual drawing of confidence intervals

6.1.2 Confidence intervals for $\mathcal{A}_{\pi\pi}$ and $\mathcal{S}_{\pi\pi}$ measurements

In this section, we treat $\mathcal{A}_{\pi\pi}$ to be independent of $\mathcal{S}_{\pi\pi}$ and vice versa. This assumption is reasonable because the correlation between $\mathcal{A}_{\pi\pi}$ and $\mathcal{S}_{\pi\pi}$ is small as described in Section 5.2.6. The Monte Carlo pseudo-experiments are also based on this assumption. The distribution for $\mathcal{A}_{\pi\pi}$ is independent of the input value of $\mathcal{S}_{\pi\pi}$ in Monte Carlo pseudo-experiments, and vise versa. We obtain the probability density function for $\mathcal{A}_{\pi\pi}$, $p(x_{A_{\pi\pi}}|\mu_{A_{\pi\pi}})$ and that for $\mathcal{S}_{\pi\pi}p(x_{S_{\pi\pi}}|\mu_{S_{\pi\pi}})$ from the distribution of CP fit result in Monte Carlo pseudo-experiments with various input values, where $\mu_{A_{\pi\pi}}$ ($\mu_{S_{\pi\pi}}$) represents the true value of $\mathcal{A}_{\pi\pi}$ ($\mathcal{S}_{\pi\pi}$), and $x_{A_{\pi\pi}}$ ($x_{S_{\pi\pi}}$) represents the measured value of $\mathcal{A}_{\pi\pi}$ ($\mathcal{S}_{\pi\pi}$). We find the distributions of fit results of $\mathcal{A}_{\pi\pi}$ and $\mathcal{S}_{\pi\pi}$ agree with the sum of the two Gaussians, and the widths of the distributions depend on the input values of Monte Carlo pseudo-experiments. Thus, we parameterize the probability density functions as follows.

$$p(x_{A_{\pi\pi}}|\mu_{A_{\pi\pi}}) = g_{A_{\pi\pi}} \cdot G(x_{A_{\pi\pi}}; b^1_{A_{\pi\pi}}, \sigma^1_{A_{\pi\pi}}) + (1 - g_{A_{\pi\pi}}) \cdot G(x_{A_{\pi\pi}}; b^2_{A_{\pi\pi}}, \sigma^2_{A_{\pi\pi}})$$
(6.5)

$$g_{A_{\pi\pi}} = \underline{g}_{A_{\pi\pi}}^{0} + \underline{g}_{A_{\pi\pi}}^{2} \cdot (\mu_{A_{\pi\pi}})^{2}$$
(6.6)

$$b_{A_{\pi\pi}}^{1} = \underline{b_{A_{\pi\pi}}^{1,0}} + \underline{b_{A_{\pi\pi}}^{1,1}} \cdot (\mu_{A_{\pi\pi}})$$
(6.7)

$$\sigma_{A_{\pi\pi}}^{1} = \underline{\sigma_{A_{\pi\pi}}^{1,0}} + \underline{\sigma_{A_{\pi\pi}}^{1,2}} \cdot (\mu_{A_{\pi\pi}})^{2}$$
(6.8)

$$b_{A_{\pi\pi}}^{2} = \underline{b}_{A_{\pi\pi}}^{2,0} + \underline{b}_{A_{\pi\pi}}^{2,1} \cdot (\mu_{A_{\pi\pi}}) + \underline{b}_{A_{\pi\pi}}^{2,2} \cdot (\mu_{A_{\pi\pi}})^{2} + \underline{b}_{A_{\pi\pi}}^{2,3} \cdot (\mu_{A_{\pi\pi}})^{3}$$

$$(6.9)$$

$$\sigma_{A_{\pi\pi}}^2 = \underline{\sigma_{A_{\pi\pi}}^{2,0}} + \underline{\sigma_{A_{\pi\pi}}^{2,2}} \cdot (\mu_{A_{\pi\pi}})^2$$
(6.10)

$$p(x_{S_{\pi\pi}}|\mu_{S_{\pi\pi}}) = g_{S_{\pi\pi}} \cdot G(x_{S_{\pi\pi}}; b_{S_{\pi\pi}}^1, \sigma_{S_{\pi\pi}}^1) + (1 - g_{S_{\pi\pi}}) \cdot G(x_{S_{\pi\pi}}; b_{S_{\pi\pi}}^2, \sigma_{S_{\pi\pi}}^2)$$

$$(6.11)$$

$$g_{S_{\pi\pi}} = \underline{g}_{S_{\pi\pi}}^{*} + \underline{g}_{S_{\pi\pi}}^{*} \cdot (\mu_{S_{\pi\pi}})$$

$$b_{G}^{1} = b_{G}^{1,0} + b_{G}^{1,1} \cdot (\mu_{S})$$
(6.12)
(6.13)

$$\sigma_{S_{\pi\pi}}^{1} = \sigma_{S_{\pi\pi}}^{1,0} + \sigma_{S_{\pi\pi}}^{1,2} \cdot (\mu_{S_{\pi\pi}})^{2}$$
(6.14)

$$b_{S_{\pi\pi}}^2 = \underline{b}_{S_{\pi\pi}}^{2,0} + \underline{b}_{S_{\pi\pi}}^{2,1} \cdot (\mu_{S_{\pi\pi}}) + \underline{b}_{S_{\pi\pi}}^{2,2} \cdot (\mu_{S_{\pi\pi}})^2 + \underline{b}_{S_{\pi\pi}}^{2,3} \cdot (\mu_{S_{\pi\pi}})^3$$
(6.15)

$$\sigma_{S_{\pi\pi}}^2 = \underline{\sigma_{S_{\pi\pi}}^{2,0}} + \underline{\sigma_{S_{\pi\pi}}^{2,2}} \cdot (\mu_{S_{\pi\pi}})^2$$
(6.16)

There are 12 parameters, indicated by the underline in above equations, for each of $\mathcal{A}_{\pi\pi}$ and $\mathcal{S}_{\pi\pi}$. The parameters are determined by an unbinned maximum likelihood fit of the distribution of CP-fit result in Monte Carlo pseudo-experiments with various input values, as listed in Table 6.1. Figures 6.2 and 6.3 demonstrate that the parameterization is reasonable.

The confidence belts obtained from this probability density functions are shown in Figure 6.4, where the vertical lines corresponding to the measured values of $\mathcal{A}_{\pi\pi}$ and $\mathcal{S}_{\pi\pi}$ in the real data are superimposed. The confidence intervals are derived from obtained confidence belts as:

$$+0.31 < \mathcal{A}_{\pi\pi} < +1.00 \qquad (90\% \text{CL}), \tag{6.17}$$

$$+0.22 < \mathcal{A}_{\pi\pi} < +1.00 \quad (95\% \text{CL}), \tag{6.18}$$

$$-1.00 < \mathcal{S}_{\pi\pi} < -0.56 \quad (90\% \text{CL}) \tag{6.19}$$

and

$$-1.00 < S_{\pi\pi} < -0.43$$
 (95%CL). (6.20)

The confidence levels to rule out the null asymmetry are calculated as:

$$\operatorname{CL}(\mu_{A_{\pi\pi}} = 0) = 0.9925$$
 (6.21)

and

$$\operatorname{CL}(\mu_{S_{\pi\pi}} = 0) = 0.9969,$$
(6.22)

which correspond to 2.7σ and 3.0σ significance, respectively.

Table 0.1. The Latanievers of $p(x_{A_{\pi\pi}} \mu_{A_{\pi\pi}})$ and $p(x_{S_{\pi\pi}} \mu_{S_{\pi\pi}})$				
$p(x_{A_{\pi\pi}} \mu_{A_{\pi\pi}})$ parameters	fit result	$p(x_{S_{\pi\pi}} \mu_{S_{\pi\pi}})$ parameters	fit result	
$g^0_{A_{\pi\pi}}$	$0.959 \ {}^+_{-} \ {}^{0.019}_{0.029}$	$g^0_{S_{\pi\pi}}$	$0.924 \ ^+_{-} \ ^{0.013}_{0.014}$	
$g^2_{A_{\pi\pi}}$	$-0.226 \ {}^+_{-} \ {}^{0.035}_{0.031}$	$g^2_{S_{\pi\pi}}$	-0.178 ± 0.022	
$b^{1,0}_{A_{\pi\pi}}$	$-0.00199 \ {}^+_{-} \ {}^{0.00089}_{0.00091}$	$b^{1,0}_{S_{\pi\pi}}$	0.0053 ± 0.0012	
$b^{1,1}_{A_{\pi\pi}}$	0.9961 ± 0.0020	$b_{S_{\pi\pi}}^{1,1}$	0.9914 ± 0.0025	
$\sigma^{1,0}_{A_{\pi\pi}}$	$0.2817 \ {}^+_{-} \ {}^{0.0017}_{0.0021}$	$\sigma^{1,0}_{S_{\pi\pi}}$	$0.3803 \ {}^+_{-} \ {}^{0.0021}_{0.0023}$	
$\sigma^{1,2}_{A_{\pi\pi}}$	-0.0471 ± 0.0037	$\sigma^{1,2}_{S_{\pi\pi}}$	-0.0515 ± 0.0045	
$b^{2,0}_{A_{\pi\pi}}$	$0.013 \ ^+_{-} \ ^{0.012}_{0.011}$	$b^{2,0}_{S_{\pi\pi}}$	0.031 ± 0.011	
$b^{2,1}_{A_{\pi\pi}}$	1.067 ± 0.023	$b_{S_{\pi\pi}}^{2,1}$	1.233 ± 0.028	
$b^{2,2}_{A_{\pi\pi}}$	$-0.010 \ ^+ \ ^0.014 \ ^0.015$	$b^{2,2}_{S_{\pi\pi}}$	-0.025 ± 0.015	
$b^{2,3}_{A_{\pi\pi}}$	$0.101 \ ^+_{-} \ ^0.029_{0.028}$	$b^{2,3}_{S_{\pi\pi}}$	$0.049 \ {}^+_{-} \ {}^{0.034}_{0.033}$	
$\sigma^{2,0}_{A_{\pi\pi}}$	$0.374 \ {}^+_{-} \ {}^{0.013}_{0.014}$	$\sigma^{2,0}_{S_{\pi\pi}}$	$0.571 \ {}^+_{-} \ {}^{0.013}_{0.012}$	
$\sigma^{2,2}_{A_{\pi\pi}}$	$-0.008 \ ^+$ $^{0.015}_{- \ 0.014}$	$\sigma^{2,2}_{S_{\pi\pi}}$	-0.020 ± 0.015	

Table 6.1: The Parameters of $p(r_A \mid \mu_A)$ and $p(r_A \mid \mu_B)$

2-dimensional confidence region for $\mathcal{A}_{\pi\pi}$ and $\mathcal{S}_{\pi\pi}$ measurements 6.1.3

In Section 6.1.2, we treat $\mathcal{A}_{\pi\pi}$ and $\mathcal{S}_{\pi\pi}$ independently. Although the correlation between $\mathcal{A}_{\pi\pi}$ and $\mathcal{S}_{\pi\pi}$ is small, the physical boundary of $\mathcal{A}_{\pi\pi}$ depends on the value of $\mathcal{S}_{\pi\pi}$ and vice versa. Strictly speaking, it is better to consider the confidence interval (region) 2-dimensionally. The probability density function $p(x_{A_{\pi\pi}}, x_{S_{\pi\pi}} | \mu_{A_{\pi\pi}}, \mu_{S_{\pi\pi}})$ is obtained from the distribution of CP-fit result in the Monte Carlo pseudo-experiments with various input values just like in Section 6.1.2. We find that the distribution of the fit result of $(\mathcal{A}_{\pi\pi}, \mathcal{S}_{\pi\pi})$ agree with the sum of the two 2-dimensional Gaussians and that the width of the distribution depend on the input values of the Monte Carlo pseudo-experiments. Thus, we parameterize the probability density function as follows.

$$p(x_{A_{\pi\pi}}, x_{S_{\pi\pi}} | \mu_{A_{\pi\pi}}, \mu_{S_{\pi\pi}}) = g \cdot G(x_{A_{\pi\pi}}; b^1_{A_{\pi\pi}}, \sigma^1_{A_{\pi\pi}}) \cdot G(x_{S_{\pi\pi}}; b^1_{S_{\pi\pi}}, \sigma^1_{S_{\pi\pi}})$$

$$(6.23)$$

$$+(1-g) \cdot G(x_{A_{\pi\pi}}; b_{A_{\pi\pi}}^2, \sigma_{A_{\pi\pi}}^2) \cdot G(x_{S_{\pi\pi}}; b_{S_{\pi\pi}}^2, \sigma_{S_{\pi\pi}}^2)$$

$$a^0 + a^{2A} \cdot (\mu_{A_{\pi\pi}})^2 + a^{2S} \cdot (\mu_{A_{\pi\pi}})^2$$
(6.24)

$$g = \underline{g}^{0} + \underline{g}^{2A} \cdot (\mu_{A_{\pi\pi}})^{2} + \underline{g}^{2S} \cdot (\mu_{S_{\pi\pi}})^{2}$$

$$b^{1}_{A_{\pi\pi}} = b^{1,0}_{A_{\pi\pi}} + b^{1,1A}_{A_{\pi\pi}} \cdot (\mu_{A_{\pi\pi}}) + b^{1,1S}_{A_{\pi\pi}} \cdot (\mu_{S_{\pi\pi}})$$

$$(6.24)$$

$$(6.25)$$

$$b_{S_{\pi\pi}}^{1} = \overline{b_{S_{\pi\pi}}^{1,0}} + \overline{b_{S_{\pi\pi}}^{1,1A}} \cdot (\mu_{A_{\pi\pi}}) + \overline{b_{S_{\pi\pi}}^{1,1S}} \cdot (\mu_{S_{\pi\pi}})$$
(6.26)

$$\sigma_{A_{\pi\pi}}^{1} = \underline{\sigma_{A_{\pi\pi}}^{1,0}} + \underline{\sigma_{A_{\pi\pi}}^{1,2A}} \cdot (\mu_{A_{\pi\pi}})^{2} + \underline{\sigma_{A_{\pi\pi}}^{1,2S}} \cdot (\mu_{S_{\pi\pi}})^{2}$$
(6.27)

(6.19)



Figure 6.2: The distribution of the $\mathcal{A}_{\pi\pi}$ fit results for Monte Carlo pseudo-experiments with the input values of $\mu_{A_{\pi\pi}} = -1,0$ and +1 (plus points) and the parameterized probability density function $p(x_{A_{\pi\pi}}|\mu_{A_{\pi\pi}})$ (solid-line curve). The dashed-line curves are indicate the second Gaussian components of the $p(x_{A_{\pi\pi}}|\mu_{A_{\pi\pi}})$.



Figure 6.3: The distribution of the $S_{\pi\pi}$ fit results for Monte Carlo pseudo-experiments with the input values of $\mu_{S_{\pi\pi}} = -1, 0$ and +1 (plus points) and the parameterized probability density function $p(x_{S_{\pi\pi}}|\mu_{S_{\pi\pi}})$ (solid-line curve). The dashed-line curves are indicate the second Gaussian components of the $p(x_{S_{\pi\pi}}|\mu_{S_{\pi\pi}})$.



Figure 6.4: The confidence belts. The horizontal axis shows the experimental value and the vertical axis shows the true parameter. Purple, red, green and blue curves represent the boundary of the confidence belt with confidence level equal to 0.683, 0.955, 0.9973 and 0.99994, respectively. The thick black vertical line shows the central value our experimental values of $\mathcal{A}_{\pi\pi}$ and $\mathcal{S}_{\pi\pi}$ in the real data.

$$\sigma_{S_{\pi\pi}}^{1} = \underline{\sigma_{S_{\pi\pi}}^{1,0}} + \underline{\sigma_{S_{\pi\pi}}^{1,2A}} \cdot (\mu_{A_{\pi\pi}})^{2} + \underline{\sigma_{S_{\pi\pi}}^{1,2S}} \cdot (\mu_{S_{\pi\pi}})^{2}$$
(6.28)

$$b_{A_{\pi\pi}}^{2} = \underline{b_{A_{\pi\pi}}^{2,0}} + \underline{b_{A_{\pi\pi}}^{2,1A}} \cdot (\mu_{A_{\pi\pi}}) + \underline{b_{A_{\pi\pi}}^{2,1S}} \cdot (\mu_{S_{\pi\pi}})$$
(6.29)

$$b_{S_{\pi\pi}}^{2} = \underline{b}_{S_{\pi\pi}}^{2,0} + \underline{b}_{S_{\pi\pi}}^{2,1A} \cdot (\mu_{A_{\pi\pi}}) + \underline{b}_{S_{\pi\pi}}^{2,1S} \cdot (\mu_{S_{\pi\pi}})$$

$$(6.30)$$

$$\sigma_{A_{\pi\pi}}^{2} = \frac{\sigma_{A_{\pi\pi}}^{2,0} + \sigma_{A_{\pi\pi}}^{2,2A} \cdot (\mu_{A_{\pi\pi}})^{2} + \sigma_{A_{\pi\pi}}^{2,2S} \cdot (\mu_{S_{\pi\pi}})^{2}}{\sigma_{A_{\pi\pi}}^{2,2A} \cdot (\mu_{S_{\pi\pi}})^{2}}$$
(6.31)

$$\sigma_{S_{\pi\pi}}^2 = \underline{\sigma_{S_{\pi\pi}}^{2,0}} + \underline{\sigma_{S_{\pi\pi}}^{2,2A}} \cdot (\mu_{A_{\pi\pi}})^2 + \underline{\sigma_{S_{\pi\pi}}^{2,2S}} \cdot (\mu_{S_{\pi\pi}})^2$$
(6.32)

The number of the parameters, indicated by the underline in above equations, is 27. The parameters are determined by an unbinned maximum likelihood fit to the distribution of the CP-fit result in the Monte Carlo pseudo-experiments with various input values, as listed in Table 6.2. Figure 6.5 demonstrates that this parameterization for the probability density functions represents the Monte Carlo pseudo-experiments well.

Figure 6.6 shows the obtained confidence region with confidence levels of 0.683, 0.955, 0.9973 and 0.99994, which correspond to 1σ , 2σ , 3σ and 4σ regions, respectively. The null-asymmetry hypothesis that $(\mu_{A_{\pi\pi}}, \mu_{S_{\pi\pi}})$ is equal to = (0,0) is ruled out with the confidence level of

$$\operatorname{CL}(\mu_{A_{\pi\pi}} = 0, \mu_{S_{\pi\pi}} = 0) = 0.9993,$$
 (6.33)

which corresponds to 3.40 σ significance. If the source of CP violation is only the B^0 - \overline{B}^0 mixing, $(S_{\pi\pi}, A_{\pi\pi}) = (-\sin 2\phi_1, 0)$ [118]. This hypothesis is ruled out with the confidence level of

$$CL(\mu_{A_{\pi\pi}} = 0, \mu_{S_{\pi\pi}} = -\sin 2\phi_1) = 0.981, \tag{6.34}$$

which corresponds to 2.34σ significance, where $\sin 2\phi_1 = 0.719$ [119].

6.2 Comparison with the Other Measurement

In this section, we review the measurement of the CP-violating asymmetry in $B^0 \to \pi^+\pi^-$ decays at the other experiment. The BABAR collaboration [120] reports the measurement of time-dependent CP-violating asymmetry in $B^0 \to \pi^+\pi^-$ decays using a data sample of 88 million $\Upsilon(4S) \to B\overline{B}$ decays collected between 1999 and 2002 with the BABAR detector at the PEP-II asymmetric-energy *B*-Factory at Stanford Linear Accelerator Center (SLAC), USA. Their results [121] are:

$$\mathcal{C}_{\pi\pi} = -0.30 \pm 0.25 (\text{stat}) \pm 0.04 (\text{syst}) \tag{6.35}$$

and

$$S_{\pi\pi} = +0.02 \pm 0.34 \text{(stat)} \pm 0.05 \text{(syst)}, \tag{6.36}$$



(d) $(\mu_{A_{\pi\pi}}, \mu_{S_{\pi\pi}}) = (+0.7, -0.7)$

Figure 6.5: The distribution of the $(\mathcal{A}_{\pi\pi}, \mathcal{S}_{\pi\pi})$ fit results for Monte Carlo pseudo-experiments (i), the parameterized probability density function $p(x_{A_{\pi\pi}}, x_{S_{\pi\pi}} | \mu_{A_{\pi\pi}}, \mu_{S_{\pi\pi}})$ (ii) and their projections onto $\mathcal{A}_{\pi\pi}$ (iii) and $\mathcal{S}_{\pi\pi}$ (iv). In projection histograms, the plus points, the solid line and the dashed line show the data in Monte Carlo pseudo-experiments, the $p(x_{A_{\pi\pi}}, x_{S_{\pi\pi}} | \mu_{A_{\pi\pi}}, \mu_{S_{\pi\pi}})$ and the second Gaussian components of the $p(x_{A_{\pi\pi}}, x_{S_{\pi\pi}} | \mu_{A_{\pi\pi}}, \mu_{S_{\pi\pi}})$, respectively.

Table 6.2: The parameters of $p(x_{A_{\pi\pi}}, x_{S_{\pi\pi}} | \mu_{A_{\pi\pi}}, \mu_{S_{\pi\pi}})$

Parameters	Fit result	Parameters	Fit result	Parameters	Fit result
g^0	$0.9515 \ {}^+_{-} \ {}^{0.0022}_{0.0023}$	g^{2A}	-0.2009 ± 0.0048	g^{2S}	-0.1669 ± 0.0046
$b^{1,0}_{A_{\pi\pi}}$	-0.00037 ± 0.00021	$b^{1,1A}_{A_{\pi\pi}}$	$0.99507 \ {}^+_{-} \ {}^{0.00048}_{0.00049}$	$b^{1,1S}_{A_{\pi\pi}}$	-0.00315 ± 0.00044
$b^{1,0}_{S_{\pi\pi}}$	0.00807 ± 0.00028	$b^{1,1A}_{S_{\pi\pi}}$	-0.00173 ± 0.00059	$b^{1,1S}_{S_{\pi\pi}}$	$0.99209 \ {}^+_{-} \ {}^{0.00070}_{0.00071}$
$\sigma^{1,0}_{A_{\pi\pi}}$	0.28149 ± 0.00032	$\sigma^{1,2A}_{A_{\pi\pi}}$	$-0.045765 \ {}^+_{-} \ {}^{0.00078}_{0.00079}$	$\sigma^{1,2S}_{A_{\pi\pi}}$	-0.013986 ± 0.00072
$\sigma^{1,0}_{S_{\pi\pi}}$	$0.38371 \ {}^+_{-} \ {}^{0.00051}_{0.00052}$	$\sigma^{1,2A}_{S_{\pi\pi}}$	-0.0373 ± 0.0011	$\sigma^{1,2S}_{S_{\pi\pi}}$	-0.0449 ± 0.0012
$b^{2,0}_{A_{\pi\pi}}$	0.00022 ± 0.00098	$b^{2,1A}_{A_{\pi\pi}}$	1.1494 ± 0.0024	$b^{2,1S}_{A_{\pi\pi}}$	0.0180 ± 0.0018
$b^{2,0}_{S_{\pi\pi}}$	0.0101 ± 0.0014	$b_{S_{\pi\pi}}^{2,1A}$	0.0140 ± 0.0023	$b_{S_{\pi\pi}}^{2,1S}$	1.3239 ± 0.0042
$\sigma^{2,0}_{A_{\pi\pi}}$	$0.3625 \ {}^+_{-} \ {}^{0.0020}_{0.0019}$	$\sigma^{2,2A}_{A_{\pi\pi}}$	0.0091 ± 0.0029	$\sigma^{2,2S}_{A_{\pi\pi}}$	-0.0198 ± 0.0029
$\sigma^{2,0}_{S_{\pi\pi}}$	0.5866 ± 0.0031	$\sigma^{2,2A}_{S_{\pi\pi}}$	-0.0621 ± 0.0042	$\sigma^{2,2S}_{S_{\pi\pi}}$	-0.0232 ± 0.0045



Figure 6.6: Confidence region for $(\mathcal{A}_{\pi\pi}, \mathcal{S}_{\pi\pi})$

where $C_{\pi\pi}$ in their notation corresponds to $-A_{\pi\pi}$ in our notation.

Discrepancies between our measurement and their measurement are 1.2σ and 2.3σ for $\mathcal{A}_{\pi\pi}$ and $\mathcal{S}_{\pi\pi}$, respectively. In order to compare the results 2-dimensionally, we construct the confidence region of their results with the same method as described in Section 6.1 assuming the Gaussian probability density function as shown in Figure 6.7(a). Figure 6.7(b) shows the comparison of the confidence regions between our results and *BABAR* results. The central values measured by *BABAR* group is included by the confidence regions corresponding to 3σ significance constructed from our results. The 2σ confidence region in our (their) results and the 1σ confidence region in their (our) results are over-wrapped. Therefore, the discrepancy between two experiments is within the statistical fluctuation.



(a) Confidence region by BABAR

(b) Confidence region by our measurement is superimposed.

Figure 6.7: Confidence region for BABAR results assuming Gaussian probability density function.

6.3 Constraint on CKM phase ϕ_2

In this section, we extract the information of ϕ_2 from our measured values of $\mathcal{A}_{\pi\pi}$ and $\mathcal{S}_{\pi\pi}$. As described in Section 2.4.5, we can extract the value of ϕ_2 from $\mathcal{A}_{\pi\pi}$ and $\mathcal{S}_{\pi\pi}$, if we know the ϕ_1 and |P/T|, which is the ratio of the amplitudes of the penguin diagram and the tree diagram. ϕ_1 is measured with a sufficient precision by *B*-factory experiments as

$$\phi_1 = 23.5^{+2.4}_{-2.2} \deg, \tag{6.37}$$

which is obtained from the average of the recent values of $\sin 2\phi_1$ from Belle [119] and BABAR [122]. On the other hand, |P/T| is estimated by the model-dependent way by M. Gronau et al. [38] as

$$|P/T| = 0.28 \pm 0.06,$$

which contains the theoretical uncertainty. Equations 2.92, 2.93 and 2.94 gives ϕ_2 information from the values of $\mathcal{A}_{\pi\pi}$ and $\mathcal{S}_{\pi\pi}$. Using these equations, we convert the confidence region in the $\mathcal{S}_{\pi\pi}$ - $\mathcal{A}_{\pi\pi}$ plane obtained in Section 6.1 into the ϕ_2 - δ plane, where δ is, roughly speaking, the strong-phase difference between the penguin diagram and the tree diagram. Figure 6.8 shows the regions for ϕ_2 and δ corresponding to the 68.3%, 95.5% and 99.73% confidence levels with given |P/T| and ϕ_1 values. Figure 6.8(c) shows the confidence regions with $\phi_1 = 23.5^{\circ}$ and |P/T| = 0.28. By comparing the Figure 6.8(a) and 6.8(f), we can see the ϕ_1 dependence at the fixed |P/T| = 0.28. We also investigate |P/T| dependence by varying |P/T| from 0.18 to 0.48 at the fixed $\phi_1 = 23.5^{\circ}$, as shown in

6.4. CONCLUSION

Figure 6.8(b), 6.8(d) and 6.8(e). The ϕ_1 dependence and |P/T| dependence of the 95.5% CL confidence region is small. The values of ϕ_2 and δ corresponding to the 95.5% CL confidence region for $A_{\pi\pi}$ and $S_{\pi\pi}$ are obtained as

 $77^{\circ} \le \phi_2 \le 151^{\circ}$

and

$$-175^{\circ} \le \delta \le -11^{\circ}$$

respectively, for the conditions, $21.3 \leq \phi_1 \leq 25.9^\circ$ and 0.18 < |P/T| < 0.48.



(f) $\phi_1 = 21.28, |P/T| = 0.28$

Figure 6.8: Confidence regions in ϕ_2 - δ plane with confidence levels of 68.3%, 95.5% and 99.73%. Dash-dotted lines indicate $\phi_2 = \pi - \phi_1$, which corresponds to $\phi_3 = 0$.

6.4 Conclusion

We have measured the *CP*-violating asymmetry in $B^0 \to \pi^+\pi^-$ decays using the 78 fb⁻¹ data sample collected at the $\Upsilon(4S)$ resonance with the Belle detector at the KEKB asymmetric-energy e^+e^- collider. We reconstruct 760

 $B^0 \to \pi^+ \pi^-$ candidates associated with the flavor information and the proper-time difference of the $B^0 \overline{B}{}^0$ pair. We perform unbinned-maximum-likelihood on the proper-time difference distribution and obtain the parameters of CP-violating asymmetry in the Standard Model as

$$A_{\pi\pi} = +0.77 \pm 0.27 (\text{stat}) \pm 0.08 (\text{syst})$$

and

 $S_{\pi\pi} = -1.23 \pm 0.41 (\text{stat}) \stackrel{+0.08}{_{-0.07}} (\text{syst}),$

where the statistical uncertainties are determined from the Monte Carlo pseudo-experiments.

We have evaluated the statistical significance with the frequentist-approach method proposed by Feldman and Cousins, and obtain the confidence region in $S_{\pi\pi}$ - $A_{\pi\pi}$ plane. Our measurement rules out the *CP*-conserving case, $A_{\pi\pi} = S_{\pi\pi} = 0$, at a 99.93% confidence level. The difference between the mesurement by *BABAR* [121] and ours is within the statistical fluctuation.

The result for $S_{\pi\pi}$ indicates that the mixing-induced *CP* violation is large, and the $A_{\pi\pi}$ term indicates the existence of the direct *CP* violation in the *B* meson decay.

Constraints within the Standard Model on the CKM angle ϕ_2 and the hadronic phase difference between the tree (T) and penguin (P) amplitudes are obtained for |P/T| values that are theoretically favored. We find an allowed region of ϕ_2 that is constrained on the unitarity triangle from other measurements.

Appendix A

Reconstruction of control samples.

In this section, we described on the event selection and reconstruction of the decay modes used as the control sample in the analysis.

We use the $B^0 \to K^+\pi^-$ event sample for the validity check of CP measurement. $B^0 \to K^+\pi^-$ decays are reconstructed as same way other than the requirement for the particle identification and ΔE . The pion tracks are required to have $L^{hh}(K;\pi) < 0.4$ and the Kaon tracks are required to have $L^{hh}(K;\pi) > 0.6$. The $B^0 \to K^+\pi^$ events are selected by requiring $-0.1121 < \Delta E < 0.0235$ GeV.

We also use the $B \to D^{*-}\pi^+$, $B \to D^-\pi^+$ and $B \to D^{*-}\rho^+$ event sample [123]. The D^{*-} candidates are reconstructed using the decay cascade, $D^{*-} \to \overline{D}{}^0\pi^-$, $\overline{D}{}^0 \to K^+\pi^-$, $K^+\pi^-\pi^0$, $K^+\pi^-\pi^-\pi^+$. The D^- candidates are reconstructed using $K^+\pi^-\pi^-$. B mesons are selected using ($\Delta E, M_{\rm bc}$). We introduce the kinematical constraints to reduce the contamination from the continuum background. The continuum background is suppressed by the requirement for the normalized second Fox-Wolfram moment, which is described in Section 4.3.2, the thrust angle $\theta_{\rm thrust}$. The selection criteria are listed in Table A.1 and A.2.

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${\cal B}$ decay mode	D decay mode	M(D)	$\Delta M \equiv m(D^{*-}) - m(\overline{D}^0)$	R_2	$ heta_{\mathrm{thrust}}$
$B^0 \to D^- \pi^+$	$D^- \to K^+ \pi^- \pi^-$	$< 2.5\sigma$	—	< 0.5	< 0.995
	$\overline{D}{}^0 \to K^+ \pi^-$	$< 10\sigma$	$< 5 \ { m MeV}/c^2$		_
$B^0 \to D^{*-} \pi^+$	$\overline{D}{}^0 \to K^+ \pi^- \pi^- \pi^0 z$	$< 3.5\sigma$	$< 3 \ { m MeV}/c^2$		< 0.98
	$\overline{D}{}^0 \to K^+ \pi^- \pi^- \pi^+$	$< 4\sigma$	$< 4 \ { m MeV}/c^2$	< 0.6	
	$\overline{D}{}^0 \to K^+ \pi^-$	$< 7\sigma$	$< 4 \text{ MeV}/c^2$	< 0.6	< 0.95
$B^0 \to D^{*-} \rho^+$	$\overline{D}{}^0 \rightarrow K^+ \pi^- \pi^- \pi^0$	$< 3.5\sigma$	$< 12 \ \mathrm{MeV}/c^2$	< 0.7	< 0.98
	$\overline{D}{}^0 \to K^+ \pi^- \pi^- \pi^+$	$< 3.5\sigma$	$< 3 \ { m MeV}/c^2$		< 0.92

Table A.1: Selection criteria for $B \to D^{*-}\pi^+$, $B \to D^-\pi^+$ and $B \to D^{*-}\rho^+$ modes.

Table A.2: Signal box for each decay mode.

Decay mode	ΔE range (GeV)	$M_{\rm bc}$ range (GeV/ c^2)
-0 -	· · · · · · · · · · · · · · · · · · ·	
$B^0 \to D^- \pi^+$	$-0.045 < \Delta E < 0.045$	$5.270 < M_{\rm bc} < 5.290$
$B^0 \rightarrow D^{*-} \pi^+$	$-0.07 < \Delta E < 0.07$	$5.270 < M_{\odot} < 5.200$
$D \rightarrow D \pi$	$-0.01 < \Delta L < 0.01$	$0.210 < M_{\rm bc} < 0.200$
$B^0 \to D^{*-} \rho^+$	$-0.05 < \Delta E < 0.08$	$5.270 < M_{\rm bc} < 5.290$

Appendix B

Measurement of the incorrect flavor-assignment probability

The wrong tag fractions for each tagging quality region, w_{tag}^l , are determined directly from the data for the six r intervals using exclusively reconstructed, self-tagged $B^0 \to D^{*-}\ell^+\nu$, $D^-\pi^+$, $D^{*-}\pi^+$, and $D^{*-}\rho^+$ decays. These samples are also used as control samples in the validation of result.

We evaluate the wrong tag fractions by reconstructing flavor-specific decays on one side, and tagging the *b*-flavor for the other side with the flavor-tagging algorithm described in Section 4.4.1 [123].

Since we know the flavors of both of B mesons in this case, we can observe the time evolution of the neutral B meson pair with the opposite flavor (OF) and the same flavor (SF), which originates from $B^0-\overline{B}^0$ mixing. The observed OF-SF asymmetry is expressed as:

$$A_{mix} = \frac{N_{OF} - N_{SF}}{N_{OF} + N_{SF}} = (1 - 2w_{\text{tag}}^l)\cos(\Delta m_d \Delta t), \tag{B.1}$$

where Δm_d is the mass-difference of the two *B* meson mass eigenstates and u_{tag}^l is the wrong tag fraction. Thus we can obtain the w_{tag}^l by measuring the amplitude of the OF-SF asymmetry.

The vertex position of the reconstructed B meson is obtained as follows. First, we obtain D vertex by vertex fit using charged K and π tracks and calculate D momentum. Then, we obtain B vertex using the pseudo-D track and remaining charged tracks (ℓ or π). The vertex position of tagging-side B meson is obtained using the same algorithm as CP eigenstate candidates described in Section 4.5.3.

We fit the Δt distribution of the SF and OF events to obtain the wrong-tag fraction, fixing Δm_d to the world average value. The PDF used for this measurement is similar to that used for the measurement of $\mathcal{A}_{\pi\pi}$ fixing $\mathcal{S}_{\pi\pi}$ to be 0. The parameters of resolution function and background shape are determined separately for modes used in this measurement.

Figure B.2 shows the measured asymmetries as a function of Δt with fit curves for the six tagging quality regions. The measured wrong-tag fractions (w_{tag}^l) are summarized in Table 4.9. It is confirmed that The flavor tagging algorithm categorize the events properly, as shown in Figure B.1.



Figure B.1: The wrong tag fraction for each tagging quality region.



Figure B.2: Asymmetries as a function of Δt with fit curves for the six r regions.

Appendix C

Detarmination of the parameters in the Δt resolution function

As described in Section 4.5.2, 4.5.3 and 5.2.2, the parameters of the detector resolution of the vertex measurement (other than the parameters related to non-primary track effect) and the outlier components are determined by the B meson lifetime measurement using hadronic B meson decays including $B^0 \to J/\psi K_S$, $B^0 \to J/\psi K^{*0}(K^{*0} \to K^-\pi^+)$, $B^0 \to D^{(*)}\pi^+$, $B^0 \to D^{*-}\rho^+$, $B^+ \to J/\psi K^+$ and $B^+ \to \overline{D}{}^0\pi^+$. The lifetimes of neutral and charged B mesons, τ_{B^0} and τ_{B^+} , and the resolution parameters are extracted simultaneously using unbinned maximum likelihood fit to the Δt distribution obtained from 78 fb⁻¹data sample [123]. The probability density function for signal events $P(\Delta t)$, expressed as

$$P(\Delta t) = (1 - f_{\rm ol}) \cdot \left[\frac{1}{2\tau_B} \exp\left(-\frac{|\Delta t|}{\tau_B}\right) \otimes R^i_{\rm sig}(\Delta t)\right] + f_{\rm ol} \cdot P_{\rm ol}(\Delta t).$$

Figure C.1(a) and C.1(b) show the lifetime fit results for neutral and charged *B* meson decays, respectively. The *B* meson lifetimes are $\tau_{B^0} = 1.551 \pm 0.0018$ (stat) and $\tau_{B^+} = 1.658 \pm 0.0016$ (stat). Both results are consistent with the world averages, $\tau_{B^0} = 1.542 \pm 0.0016$ and $\tau_{B^+} = 1.674 \pm 0.0018$ [42].



Figure C.1: The Δt distribution for the hadronic *B* decays in 78 fb⁻¹sample. The open circles and solid lines show the data and the fit results, respectively. The dashed lines show the contributions from the background events. The dotted lines show the outlier components.

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